UNIVERSIDADE da MADEIRA

Continuum Mechanics

Series of exercises 4 - Stress

1. Let the components of the stress tensor at P be given in matrix form by

$$\left[\widehat{T}\right] = \begin{bmatrix} 21 & -63 & 42 \\ -63 & 0 & 84 \\ 42 & 84 & -21 \end{bmatrix}$$

   in units of mega-Pascals (MPa). Determine

   (a) the stress vector on the plane at P having the unit normal $\mathbf{n} = \frac{1}{7}(2\mathbf{e}_1 - 3\mathbf{e}_2 + 6\mathbf{e}_3)$.

   (b) the stress vector on a plane at P parallel to the plane ABC, with $A = (1, 0, 0)\,; B = (0, 1, 0)\,; C = (0, 0, 2)$.

2. The components of the stress tensor at P are given in MPa with respect to axes $x_1x_2x_3$ by the matrix

$$[T_{ij}] = \begin{bmatrix} 57 & 0 & 24 \\ 0 & 50 & 0 \\ 24 & 0 & 43 \end{bmatrix}$$

   Determine the principal stresses and the principal stress directions at P.

3. The stress matrix in MPa when referred to axes $x_1x_2x_3$ is

$$\left[\widehat{T}\right] = \begin{bmatrix} 3 & -10 & 0 \\ -10 & 0 & 30 \\ 0 & 30 & -27 \end{bmatrix}$$

   Determine the principal stresses and the principal stress directions.

4. The stress tensor at P is given with respect to $x_1x_2x_3$ in matrix form with units of MPa by

$$[T_{ij}] = \begin{bmatrix} 4 & b & b \\ b & 7 & 2 \\ b & 2 & 4 \end{bmatrix}$$

   where b is unspecified. Knowing the following relations for the principal stress values: $T_3 = 3\,\text{MPa}; T_1 = 2T_2$. Determine:

   (a) the principal stress values

   (b) the value of b

   (c) the principal stress direction of T_2.

5. The state of stress at point P is given in MPa with respect to axes $x_1x_2x_3$ by the matrix

$$\left[\widehat{T}\right] = \begin{bmatrix} 25 & 0 & 0 \\ 0 & -30 & -60 \\ 0 & -60 & 5 \end{bmatrix}$$

(a) Determine the stress vector on the plane whose unit normal is $\mathbf{n} = \frac{1}{3}(2\mathbf{e}_1 + \mathbf{e}_2 + 2\mathbf{e}_3)$.

(b) Determine the normal stress component and shear component on the same plane.

6. If the state of stress at a point is

$$\left[\widehat{T}\right] = \begin{bmatrix} 300 & 0 & 0 \\ 0 & -200 & 0 \\ 0 & 0 & 400 \end{bmatrix} \text{ kPa}$$

find:

(a) the magnitude of the shearing stress on the plane whose normal is in the direction of $2\mathbf{e}_1 + 2\mathbf{e}_2 + \mathbf{e}_3$.

(b) the maximum shearing stress.

7. Suppose the body force vector is $\mathbf{B} = -g\mathbf{e}_3$, where g is a constant. Consider the following stress tensor

$$\left[\widehat{T}\right] = \alpha \begin{bmatrix} x_2 & -x_3 & 0 \\ -x_3 & 0 & -x_2 \\ 0 & -x_2 & T_{33} \end{bmatrix}$$

and find an expression for T_{33} such that $\widehat{T}$ satisfies the equations of equilibrium.

8. Given the following stress distribution

$$\left[\widehat{T}\right] = \begin{bmatrix} x_1 + x_2 & T_{12}(x_1, x_2) & 0 \\ T_{12}(x_1, x_2) & x_1 - 2x_2 & 0 \\ 0 & 0 & x_2 \end{bmatrix}$$

find T_{12} so that the stress distribution is in equilibrium with zero body force and so that the stress vector on $x_1 = 1$ is given by $\mathbf{t} = (1 + x_2)\,\mathbf{e}_1 + (5 - x_2)\,\mathbf{e}_2$.

9. The stress tensor at P relative to axes $x_1x_2x_3$ has components in MPa given by the matrix representation

$$\left[\widehat{T}\right] = \begin{bmatrix} T_{11} & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

where T_{11} is unspecified. Determine the direction $\mathbf{n}$ at P for which the plane perpendicular to $\mathbf{n}$ will be stress-free, that is, for which $\mathbf{t_n} = 0$ on that plane. What is the required value of T_{11} for this condition?

10. With respect to axes $x_1x_2x_3$ the stress state is given in terms of the coordinates by the matrix

$$\left[\widehat{T}\right] = \begin{bmatrix} x_1x_2 & x_2^2 & 0 \\ x_2^2 & x_2x_3 & x_3^2 \\ 0 & x_3^2 & x_3x_1 \end{bmatrix}$$

Determine:

(a) the body force components as functions of the coordinates if the equilibrium equations are to be satisfied everywhere.

(b) the stress vector at point $P\,(1, 2, 3)$ on the plane whose outward unit normal makes equal angles with the positive coordinate axes.

11. Relative to the Cartesian axes $x_1 x_2 x_3$ a stress field is given by the matrix

$$\left[\widehat{T}\right] = \begin{bmatrix} \left(1 - x_1^2\right) x_2 + \frac{2}{3} x_2^3 & -\left(4 - x_2^2\right) x_1 & 0 \\ -\left(4 - x_2^2\right) x_1 & -\frac{1}{3}\left(x_2^3 - 12 x_2\right) & 0 \\ 0 & 0 & \left(3 - x_1^2\right) x_2 \end{bmatrix}$$

(a) Show that the equilibrium equations are satisfied everywhere for zero body forces.

(b) Determine the stress vector at the point $P\,(2, -1, 6)$ of the plane whose equation is $3x_1 + 6x_2 + 2x_3 = 12$.

Solutions:

1a) $\mathbf{t_n} = (69, 54, -42)$ MPa; b) $\mathbf{t_n} = (-14, -14, 77)$ MPa; 2) $T_1 = 25\,\text{MPa}, T_2 = 50\,\text{MPa}, T_3 = 75\,\text{MPa}; \mathbf{n}_1 = \left(\pm\frac{3}{5}, 0, \mp\frac{4}{5}\right), \mathbf{n}_2 = (0, \pm 1, 0), \mathbf{n}_3 = \left(\pm\frac{4}{5}, 0, \pm\frac{3}{5}\right)$; 3 $T_1 = 0\,\text{MPa}, T_2 = 23\,\text{MPa}, T_3 = -47\,\text{MPa}; \mathbf{n}_1 = (\pm 0.912, \pm 0.274, \pm 0.304)$; 4a) $T_1 = 8\,\text{MPa}, T_2 = 4\,\text{MPa}, T_3 = 3\,\text{MPa}$; b) $b = 0$; c) $\mathbf{n}_2 = (\pm 1, 0, 0)$; 5a) $\mathbf{t_n} = \frac{50}{3}(1, -3, -1)$ MPa; b) $T_N \approx -16,67\,\text{MPa}, T_S \approx 52,70\,\text{MPa}$; 6a) $T_S \approx 260,10\,\text{kPa}$; b) $T_{Smax} = 300\,\text{kPa}$; 7) $T_{33} = \left(\frac{\rho g}{\alpha} + 1\right) x_3 + f(x_1, x_2)$; 8) $T_{12} = 2x_1 - x_2 + 3$; 9) $\mathbf{n} = \frac{2}{3}\left(1, -\frac{1}{2}, -1\right), T_{11} = 2\,\text{MPa}$; 10a) $\mathbf{B} = -\frac{1}{\rho}(3x_2, 3x_3, x_1)$; b) $\mathbf{t_n} = \frac{1}{\sqrt{3}}(6, 19, 12)$; 11b) $\mathbf{t_n} = \frac{1}{7}(-29, -40, 2)$.