UNIVERSIDADE da MADEIRA

Mecânica dos Meios Contínuos

Série de exercícios 7 - Fluido viscoso de Newton

1. A glass of water moves vertically upward with a constant acceleration **a**. Find the pressure at a point whose depth from the surface of the water is h.

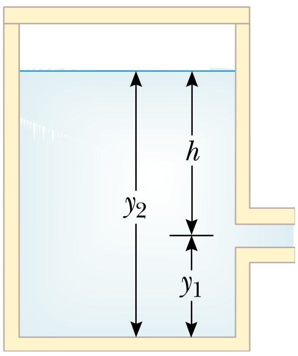
2. In astrophysical applications, an atmosphere having the relation between the density ρ and pressure p given by
$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^n$$
where p_0 and ρ_0 are some reference pressure and density, is known as a polytropic atmosphere. Find the distribution of pressure and density in a polytropic atmosphere.

3. Assuming air is an ideal gas whose temperature varies linearly with altitude as $T = T_0 - \alpha x_3$ where T_0 is ground level temperature and x_3 measures height above the earth, determine the air pressure in the atmosphere as a function of x_3 under hydrostatic conditions.

4. A barotropic fluid having the equation of state $p = \lambda \rho^k$ where λ and k are constants is at rest in a gravity field in the x_3 direction. Determine the pressure in the fluid with respect to x_3 and p_0, the pressure at $x_3 = 0$.

5. Blood flows from an artery of radius 0.3 cm, where its speed is $10\,\mathrm{cm\,s^{-1}}$, into a region where the radius has been reduced to 0.2 cm because of thickening of the arterial walls (arteriosclerosis). What is the speed of the blood in the narrower region?

6. An enclosed tank containing a liquid of density ρ has a hole in its side at a distance y_1 from the tank's bottom (see figure). The hole is open to the atmosphere, and its diameter is much smaller than the diameter of the tank. The air above the liquid is maintained at a pressure p. Determine the speed of the liquid as it leaves the hole when the liquid's level is a distance h above the hole.



7. Water is moving with a speed of $5.0\,\mathrm{m\,s^{-1}}$ through a pipe with a cross-sectional area of $4.0\,\mathrm{cm^2}$. The water gradually descends 10 m as the pipe cross-sectional area increases to $8.0\,\mathrm{cm^2}$.

   (a) What is the speed at the lower level?

   (b) If the pressure at the upper level is 1.5×10^5 Pa, what is the pressure at the lower level?

8. Given
$$\varphi = x^3 - 3xy^2$$

   (a) Show that φ satisfies the Laplace equation.

(b) Find the irrotational velocity field.

(c) Find the pressure distribution for an incompressible homogeneous fluid, if at $(0,0,0)$ $p = p_0$ and $\Omega = gz$.

Solutions:

1) $p = p_0 + \rho\,(a+g)\,h$; 2) If $n = 1$: $p = p_0 \exp\left(\frac{-\rho_0 g(z-z_0)}{p_0}\right)$, $\rho = \rho_0 \exp\left(-\frac{1}{n}\frac{\rho_0 g(z-z_0)}{p_0}\right)$; if $n \neq 1$: $p = \left(p_0^{\frac{n-1}{n}} + \frac{n-1}{n}\left(-\rho_0 g p_0^{\frac{-1}{n}}\,(z-z_0)\right)\right)^{\frac{n}{n-1}}$, $\rho = \rho_0 \frac{\left(p_0^{\frac{n-1}{n}} + \frac{n-1}{n}\left(-\rho_0 g p_0^{\frac{-1}{n}}\,(z-z_0)\right)\right)^{\frac{1}{n-1}}}{p_0^{\frac{1}{n}}}$; 3) $p = p_0\left(\frac{T_0 - \alpha x_3}{T_0}\right)^{\frac{Mg}{R\alpha}}$; 4) If $k = 1$: $p = p_0 \exp\left(-\frac{g x_3}{\lambda}\right)$; if $k \neq 1$: $p = \left(-\frac{k-1}{k}\frac{g x_3}{\lambda^{\frac{1}{k}}} + p_0^{\frac{k-1}{k}}\right)^{\frac{k}{k-1}}$; 5) $v_2 = 22.5\,\frac{\mathrm{cm}}{\mathrm{s}}$; 6) $v_1 = \sqrt{\frac{2}{\rho}\,(p-p_0) + 2gh}$; 7a) $v_2 = 2.5\,\mathrm{m/s}$; 7b) $p_2 \approx 2.6 \times 10^5\,\mathrm{Pa}$; 8b) $\mathbf{v} = (3x^2 - 3y^2, -6xy, 0)$; 8c) $p = p_0 - \frac{\rho v^2}{2} - \rho g z = p_0 - \frac{\rho}{2}\left((3x^2-3y^2)^2 + (6xy)^2\right) - \rho g z$.