hydraulic (points 5) manometers is 5 and 2%, respectively, and is shown in the figure by the vertical bars. The results of the experiment agree satisfactorily with the results of the calculation. We mention that the experimental data were obtained on a model with an inclination of the generator, i.e., the generators of the nose and base cones were joined without using an arc of a circle.

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ELECTRIC CHARACTERISTICS OF A PROBE IN A SUBSONIC PLASMA FLOW

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Lam [1] and Su [2] have formulated and given some results of the solution to the problem of the concentration distributions of the charged particles and electric field in a weakly ionized plasma that flows past a conducting body (an electric probe) under the condition that the Reynolds number of the oncoming flow is high. In the present paper, this problem is solved by the method of exterior and interior asymptotic expansions with respect to a small parameter [3]. The form of the current—voltage characteristics of the probe is found as a function of the determining parameters of the problem. Data of an experimental verification of the obtained results for the case of a cold probe in a flowing air plasma containing added potassium are given.

1. Formulation of the Problem

We consider the steady flow of a multicomponent moderately ionized thermally equilibrium plasma containing neutral components, positive singly charged ions of one species, and electrons flowing past a conducting body (an electric probe). We shall assume that the mean free path of the charged particles is much less than all the characteristic macroscopic dimensions of the problem, including the thickness of the Debye boundary layer. We shall assume that the mole concentrations of the neutral components of the plasma are constant and given. Also assuming given the density distribution, the mass-average
velocity, and the temperature of the plasma, to find the fields of the mole concentrations of the ions and the electrons, \(x_i\) and \(x_e\), and the electric potential \(\phi\) we have the boundary-value problem [4]

\[
\begin{align*}
\n \sum_{j=1}^{n} J_j &= 0, \\
\n \sum_{j=1}^{n} n_j v_j \nabla n_j &= \sum_{k=1}^{M} \frac{1}{kT} \frac{\partial n_k}{\partial x_k}, \\
\n \sum_{j=1}^{n} \nabla (n_j v_j) &= 0, \\
\n \nabla \cdot \mathbf{J} &= 0.
\end{align*}
\]

On the surface of the probe and far from it, respectively,

\[
\begin{align*}
\n x_i &= x_e = 0, & q^o &= q_w^o, & \phi &= 0.
\end{align*}
\]

Here, \(n, T, v^o\) are the concentration, temperature, and mass-average velocity of the plasma, \(v^o\) is the gradient operator, \(J_i\) and \(J_e\) are the number densities of the diffusion fluxes of the ions and electrons, \(e\) is the electron charge, \(k\) is Boltzmann’s constant, \(a_{jk}\) are resistance coefficients, \(\xi\) is the electron charge, \(k\) is Boltzmann’s constant, \(a_{jk}\) are resistance coefficients, \(W\) is the rate of change of the concentrations of the ions and the electrons due to ionization and recombination, \(x_k\) \((k = 1, \ldots, M)\) and \(M\) are the mole concentrations of the neutral components of the plasma and the number of these components, and \(\phi_w\) and \(x_w\) are the surface potential of the probe and the mole concentration of the charged particles in the undisturbed plasma (given quantities).

The transport equations (1.1) and (1.2) are obtained from the general Stefan–Maxwell relations [5] when thermodiffusion, barodiffusion, and the diffusion velocities of the neutral components are ignored. In contrast to the transport equations used in [4], these equations take into account the ion–electron collisions (the last terms on the right-hand sides).

We shall assume that the charged particle–neutral resistance coefficients \(a_{jk}\) \((j = i, e; k = 1, \ldots, M)\) depend only on the pressure and temperature of the plasma, and that the ion–electron resistance coefficient \(a_{ie}\) also depends on the concentrations of the charged particles. We note that when the resistance coefficients are calculated in the first approximation in the expansion with respect to Sonin polynomials in the Chapman–Enskog theory this assumption is rigorously satisfied [6].

Eliminating from Eqs. (1.1)-(1.3) the diffusion fluxes and going over to dimensionless variables, we write the problem (1.1)-(1.6) in the form

\[
\begin{align*}
\n \sum_{j=1}^{n} J_j &= 0, \\
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\end{align*}
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\end{align*}
\]

On the surface of the probe and far from it we have, respectively,

\[
\begin{align*}
\n x_i &= x_e = 0, & \phi &= \phi_w, & \phi &= 0.
\end{align*}
\]
Here, $W_0$ is the characteristic rate of generation of charged particles in unit volume due to ionization or the rate of disappearance due to recombination, $L$ is the characteristic dimension of the probe, and the index $\infty$ is appended to the corresponding quantities far from the probe. In writing down the system of equations, we have assumed that the free stream Mach number is fairly small and the pressure field of the plasma homogeneous, that the ion-neutral and electron-neutral resistance coefficients change in the same way in the considered range of variation of the plasma temperature, and that the ratio $\beta$ of the diffusion coefficients in the plasma is constant [7].

The problem (1.7)-(1.11) contains as coefficients the given functions $a(0), \theta(\mathbf{r}; \Re), v(\mathbf{r}; \Re)$ and the given parameters $\nabla, \Re, \chi, \varepsilon, \psi_\infty$ ($\mathbf{r}$ is the radius vector of the instantaneous point; it is assumed that the Schmidt number, calculated using the ion diffusion coefficient, is of order unity and that the parameter $\Re$ introduced above is in order of magnitude equal to the ordinary gas-dynamic Reynolds number). In the general case, the formulation of the problem must also be augmented by the expressions for the functions $b$ and $W$:

$$b=b(0, z_1, z_2), \quad W=W(0, z_1, z_2).$$

2. Asymptotic Solution

We shall assume that the parameters $\Re, \chi^{-1}, \varepsilon^{-1}, \psi_\infty$ are large, and we establish between them the following order of magnitude relations: $\chi^{-1}/\Re \sim k_1; \varepsilon^{-1}/\Re \sim k_2; \psi_\infty/\Re \sim k_3$ ($k_1, k_2, k_3, s$ are given constants). We shall assume that the parameter $\beta$ is fixed.

The exterior asymptotic expansion of the solution of the problem (1.7)-(1.11), valid in the region of the inviscid flow, is associated with the limit $\Re \to \infty, \chi \to 0, \varepsilon \to 0, \psi_\infty \to \infty, \mathbf{r}$ fixed. We shall seek this expansion in the form

$$z_i=g_i(r)+\alpha g_2(r)+\ldots, \quad z_i=f_i(r)+\alpha f_2(r)+\ldots, \quad q=Re^{\psi_i}(r)+\ln \omega^{-1}q_1(r)+q_2(r)+\ldots, \quad \omega=\varepsilon Re \quad (2.1)$$

Here, $\alpha$ is an unknown small parameter, which depends on $\Re, \chi, \varepsilon, \psi_\infty$. We note that in the general case the expansion of the potential may contain not only the term of order $\ln \omega^{-1}$ but also other terms between the terms of order $Re^{1/2}$ and order unity. However, it can be shown that even when these terms are taken into account all the following remains valid.

In the considered limit, we have the expansions

$$\theta(r; \Re)=1+o(Re^{-\psi}), \quad v(r; \Re)=-v_i(r)+\ldots, \quad a(0)=1+o(Re^{-\psi}) \quad (2.2)$$

Substituting the expansions (2.1) and (2.2) in Eqs. (1.7)-(1.9) and the first boundary condition (1.11), we obtain a problem for the functions $g_1$ and $f_1$:

$$v_i \nabla g_i=k_i W_i, \quad v_i \nabla f_i=k_i W_i, \quad g_i=f_i, \quad \mid \mathbf{r} \mid \to \infty, \quad g_i=1, \quad f_i=1, \quad W_i=W(1, g_i, f_i)$$

By virtue of the obvious equation $W(1, 1, 1) = 0$, we find

$$g_i=f_i=1$$

To derive the determining equation for the function $\psi_1$, we subtract (1.7) from (1.8), transform the difference of the convective terms by means of (1.9), and substitute the expansions (2.1) and (2.2) in the obtained equation; the boundary condition far from the probe is the second condition (1.11):

$$v_i^2 \psi_i=0, \quad \mid \mathbf{r} \mid \to \infty, \quad \psi_i \to 0 \quad (2.3)$$

It is easy to show that $\omega=\varepsilon Re^{-\psi}$. Then for the functions $\psi_2$ and $\psi_3$ we also obtain the Laplace equation with the conditions (2.3) of damping at infinity.

We now consider the first interior expansion, which is valid in the quasineutral part of the gas-dynamic boundary layer. To simplify the exposition, we shall assume that the problem is axisymmetric. The required expansion is associated with the limit

$$\Re \to \infty, \quad \chi \to 0, \quad \varepsilon \to 0, \quad \psi_\infty \to \infty, \quad y_1=y Re^{\psi} \to \infty$$

fixed (the dimensionless curvilinear coordinates $x, y$ are measured along the generator of the surface of the probe and along the normal to it, respectively) and has the form
where the subscript \( w \) is appended to the values of the corresponding functions on the surface of the probe.

In the considered limit,
\[ \theta_0 = \theta_0(x, y_0) + \ldots, \quad \nu_x = \nu_x(x, y_0) + \ldots, \quad \nu_y = \nu_y(x, y_0) + \ldots \]

For the functions \( g_{i}, f_{i}, \varphi_{i} \), we obtain the system of equations
\[
\begin{align*}
\frac{1}{\theta_i} \left( v_{ni} \frac{\partial g_{i}}{\partial x} + v_{ni} \frac{\partial g_{i}}{\partial y} \right) - \frac{\partial}{\partial y} \left( \frac{(1+b_{i})a_{i}}{1+b_{i}} \frac{\partial g_{i}}{\partial y} + b_{i} \frac{\partial f_{i}}{\partial y} \right) &= \frac{g_{i}}{\theta_i} W_{i} (2.4) \\
\frac{4}{\theta_i} \left( v_{ni} \frac{\partial f_{i}}{\partial x} + v_{ni} \frac{\partial f_{i}}{\partial y} \right) - \frac{\partial}{\partial y} \left( \frac{(1+b_{i})a_{i}}{1+b_{i}} \frac{\partial f_{i}}{\partial y} + b_{i} \frac{\partial g_{i}}{\partial y} \right) &= \frac{f_{i}}{\theta_i} W_{i} (2.5) \\
g_{i} = f_{i}, \quad a_{i} = a(0), \quad b_{i} = b(0), \quad f_{i}, \varphi_{i}, \quad W_{i} = W(0, g_{i}, f_{i}) (2.6)
\end{align*}
\]

The boundary conditions as \( y_4 \to \infty \) are the conditions of matching to the exterior expansion \( (2.1) \), and the boundary conditions on the wall are the first conditions in \((1.10)\):
\[
\begin{align*}
y_4 &= 0, \quad g_{i} = f_{i} = 0; \quad y_4 \to \infty, \quad g_{i} \to 1, \quad f_{i} \to 1 \\
y_4 \to \infty, \quad \varphi_{i} &= \varphi_{i}(\partial \psi_{i}/\partial y) = \psi_{i} + \varphi_{i} \psi_{i}(x) + \ldots (2.7)
\end{align*}
\]

Combining Eqs. \((2.4)\) and \((2.5)\), we obtain for the function \( g_{4} \) the ambipolar diffusion equation
\[
\frac{1}{\theta_{4}} \left( v_{n4} \frac{\partial g_{4}}{\partial x} + v_{n4} \frac{\partial g_{4}}{\partial y} \right) - \frac{2}{1+\beta} \frac{\partial}{\partial y} \left( a_{4} \frac{\partial g_{4}}{\partial y} \right) = \frac{g_{4}}{\theta_{4}} W_{4} (2.9)
\]

For the function \( \varphi_{4} \), we find the expression
\[
\varphi_{4} = \int \left\{ \left[ \frac{g_{4}^{-1} + (1+\beta)\beta_{4}}{1+(1+\beta)\beta_{4}} \frac{\partial}{\partial y} \right] a_{4} - 1 \right\} \left( \frac{\partial \psi_{4}}{\partial y} \right) + \theta_{4} \frac{1-\beta}{1+\beta} \frac{\partial \ln \psi_{4}}{\partial y} \right) dy_{4} + \left( \frac{\partial \psi_{4}}{\partial y} \right) y_{4} + \varphi_{4}(x) (2.10)
\]

Since this expression has a singularity on the wall, it is necessary to consider the second interior expansion, which is valid in the Debye boundary layer at the wall. This expansion is associated with the limit \( Re \to \infty, \chi \to 0, \epsilon \to 0, \varphi_{w} \to \infty, x, y_{i} = y/(\epsilon Re^{-\nu_{i}}) \geq 0 \) fixed and has the form
\[
\begin{align*}
z_{i} &= \omega_{i} g_{i}(x, y_{i}) + \ldots, \quad z_{i} = \omega_{i} f_{i}(x, y_{i}) + \ldots \\
\varphi &= \Re^{\nu_{i}} \ln \omega^{-1} \left[ \varphi_{w}(x) - \frac{u_{i}(x)}{3} \right] + \varphi_{0}(x, y_{i}) + \ldots \\
u_{i}(x) &= \theta_{o} \left[ \frac{1-\beta}{1+\beta} + \frac{u_{i-1}(x) \epsilon_{i-1}^{\nu_{i}}}{1+(1+\beta)\beta_{i}} \left( \frac{\partial \psi_{i}}{\partial y} \right) \right], \quad u_{i}(x) = \left( \frac{\partial g_{i}}{\partial y} \right) (2.11)
\end{align*}
\]

In this limit,
\[
\begin{align*}
\theta &= \theta_{o} + \ldots, \quad \nu_{x} = \omega_{i} \left( \frac{\partial v_{x}}{\partial y} \right) (x) y_{i} + \ldots, \quad \nu_{y} = \omega_{i} \Re^{\nu_{i}} \left( \frac{\partial^{2} v_{x}}{\partial y^{2}} \right) (x) y_{i}^{2} + \ldots
\end{align*}
\]

After a change of variables, we obtain the following one-parameter boundary-value problem:
\[
\begin{align*}
\frac{\partial}{\partial y_{0}} \left( \frac{\partial g_{e}}{\partial y_{0}} + g_{e} \frac{\partial \psi}{\partial y_{0}} \right) &= 0, \quad \frac{\partial}{\partial y_{0}} \left( \frac{\partial f_{e}}{\partial y_{0}} - f_{e} \frac{\partial \psi}{\partial y_{0}} \right) &= 0, \quad \frac{\partial^{2} \psi}{\partial y_{0}^{2}} = f_{e} - g_{e} (2.12)
\end{align*}
\]

\[
\begin{align*}
y_{e} &= 0, \quad g_{e} = f_{e} = 0; \quad y_{e} \to \infty, \quad g_{e} \approx y_{e} + \ldots, f_{e} \approx y_{e} + \ldots, \Psi \approx \psi_{e}^{\nu_{i}} (1) \\
\psi_{i} &= u_{i-1}(x) y_{i}^{\nu_{i}}, \quad g_{i} = [u_{i-1}(x)]^{-\nu_{i}} g_{e}, \quad f_{i} = [u_{i-1}(x)]^{-\nu_{i}} f_{e} \\
\Psi &= \psi_{e}^{\nu_{i}} (2.13)
\end{align*}
\]
The obtained problem is identical to the problem of the Debye layer in a weakly ionized, chemically frozen plasma at rest, for which a numerical solution is given in [8]. To accuracy $o(1)$, we find for $\varphi_w$ the expression (the function $\varphi_w(\lambda)$ is calculated in [8])

$$\varphi_w = \text{Re}\psi_{1w}(x) + \ln \omega^{-1} \psi_{2w}(x) + \psi_{1w}(x) - \ln \omega^{-1} \frac{u_i(x)}{3} + \frac{2u_t(x)}{3} \ln \left[ \theta_{u_t(x)} + \theta_{u}(x) \right],$$

The first three terms of the right-hand side of this expression can be interpreted as the contribution of the region of inviscid flow to the total probe-plasma potential difference; the following three terms, as the contribution of the quasineutral part of the gas-dynamic boundary layer; and the last term, as the contribution of the Debye boundary layer.

We can now formulate the following algorithm for constructing the complete solution to the problem (1.7)-(1.11). As was shown above, the concentrations of the charged particles in the region of the inviscid flow are constant in the first approximation:

$$z_i = 1 + o(1), \quad z_t = 1 + o(1).$$

In the quasineutral part of the gas-dynamic boundary layer, the concentrations of the charged particles can be determined from the solution of the problem (2.9), (2.7):

$$z_i = g_i + o(1), \quad z_t = g_t + o(1).$$

We then calculate the auxiliary function $q_1$, which satisfies the Laplace equation and the condition of damping far from the probe and is equal to $\varphi_w$ on the surface of the probe. We then calculate the functions $u_1(x)$ and $u_2(x)$, replacing the function $\varphi_i$ on the right-hand side of the expressions (2.11) and (2.14) by $\text{Re}^{-\frac{1}{2}}q_1$. The distribution of the potential in the region of the inviscid flow is determined from the solution of the Dirichlet problem (2.16)-(2.18) given below;

$$q_i = q_0 + o(1),$$

$$V^2q_i = 0,$$

On the surface of the probe and far from it we have, respectively,

$$q_s = q_s + \ln \omega^{-1} \frac{u_i(x)}{3} - \frac{2u_t(x)}{3} \ln \left[ \theta_{u_t(x)} + \theta_{u}(x) \right].$$

In the quasineutral part of the gas-dynamic boundary layer, the distribution of the potential can be calculated in accordance with

$$q_s = q_2 + \frac{\sum}{\omega} \left\{ \left[ \frac{g_i^{-1} + (1+\beta)b}{\theta_{u_i} - 1} \right] \text{Re}^{-\beta} \left( \frac{\partial g_i}{\partial y} \right) + \theta_i \frac{1-\beta}{1+\beta} \left( \frac{\partial g_i}{\partial y} \right) \right\} dy_i + o(1)$$

Finally, in the Debye boundary layer the distributions of the concentrations of the charged particles and the potential can be determined by solving the problem (2.12):

$$z_i^p = \omega^5 \left[ \theta_{u_t(x)} \right]^{\gamma_{1i}} g_i + o(\omega^5), \quad z_s^p = \omega^5 \left[ \theta_{u_t(x)} \right]^{\gamma_{1i}} g_t + o(\omega^5), \quad \varphi^p = \varphi_w + o(\Psi - \Psi_w) + o(1)$$

From the physical point of view, our method of calculating the distribution of the potential has the following meaning. The main contribution to the total probe-plasma potential difference is made by the region of inviscid flow. Therefore, in the first approximation the distribution of the potential in this region and the distribution of the current density on the surface of the probe can be determined without taking into account the voltage drop across the boundary layer. Then, from the distribution of the
current density determined in this way we can calculate these voltage differences, after
which the distribution of the potential in the region of the inviscid flow can be de-
termined more accurately.

It is important to emphasize that the algorithm found above makes it possible to
construct the solution directly for given surface potential of the probe, in contrast
to the analogous algorithm in [1], in the framework of which the solution is constructed
for given distribution of the current density on the surface of the probe, and the
potential of the surface is found from the solution, being in the general case non-
constant along the surface.

Our analysis is valid provided that on the complete surface of the probe the
Debye layer is homogeneous and can be described in the framework of a single asymptotic
expansion. This condition is satisfied if on the complete surface of the probe the
function $\varphi_{w}^{-1}u_1(x)$, which is a parameter of the problem (2.12), does not exceed unity
in modulus [8]. This last condition is satisfied when

$$-\varphi_w < \varphi < \varphi_w, \quad \varphi_c = \frac{1 + (1 + \beta)b_w}{1 + \beta} 2a_w Re^w \min_x \left[ -\frac{u_1(x)}{u_1(x)} \right], \quad u_1(x) = \left( \frac{\partial \varphi}{\partial y} \right)_w \quad (2.19)$$

Here, $\varphi_3$ is a harmonic function equal to unity on the surface of the probe and
vanishing at infinity; it is assumed that on the entire surface of the probe $(\partial \varphi_3/\partial y)_w$
< 0.

3. Current—Voltage Characteristic

The current density $j^o$ and the integrated electric current $I^o$ from the surface of
the probe into the plasma can be expressed as follows in terms of the distribution of
the potential in the region of inviscid flow:

$$j = -\left( \frac{\partial \varphi^i}{\partial y} \right)_w + O(1 + \epsilon Re^w), \quad I = -\int_x \left( \frac{\partial \varphi^i}{\partial y} \right)_w dS + O(1) \quad (3.1)$$

$$j^o = \frac{kT^o}{eL} \sigma_{oj}, \quad I^o = \frac{kT^o L \sigma_{w}}{e} I, \quad \sigma_w = \frac{n_o x_o e^2 D_{eo}(1 + \beta)}{kT_w [1 + (1 + \beta)b_w]}$$

where $dS$ is the dimensionless element of area of the surface of the probe, $\sigma_w$ is the
conductivity of the undisturbed plasma determined in the usual manner (the formula
written down for $\sigma_w$ can be obtained from Eqs. (1.1) and (1.2)), and the integral is
taken over the complete surface $\Sigma$ of the probe.

Substituting (2.15) in the second expression (3.1), we obtain for the current—
voltage characteristic the expression

$$I = -\int_x \left( \frac{\partial \varphi^i}{\partial y} \right)_w dS + O(1) \quad (3.2)$$

Retaining only the first term in the expansion (2.15), we can obtain the simpler
expression

$$I = \frac{4\pi C}{L} \varphi_w \left[ 1 + O \left( \frac{1}{Re^w} \right) \right], \quad C = -\frac{L}{4\pi} \int_x u_1(x) dS \quad (3.3)$$

The coefficient $C$ with the dimensions of length introduced here has the meaning
of the electrostatic capacity of the probe.

The obtained relations describe the current—voltage characteristic of the probe
over the interval $(-\beta \varphi_C^*, \varphi_C^*)$; as can be seen from (3.3), the characteristic is nearly
straight in this interval.

We consider the current—voltage characteristic for $\varphi_w \gg \varphi_C^*$, or, which is the same
thing, for $L > L_o = 4n_o CL^{-1} \varphi_C$. Leaving aside a rigorous analysis, we give some qualitative
arguments that establish the possible form of the characteristic in this region. The
form of the current—voltage characteristic for $\varphi_w \ll -\beta \varphi_C$ can be considered similarly.

We introduce the value $I_{ES}$ of the greatest current flowing from the probe into
the plasma in the case when the Debye layer on the complete surface of the probe is asymptotically thin (compared with the thickness of the gas-dynamic boundary layer, which has the order Re<sup>-1/2</sup>). In the first approximation, this quantity is given by [7]

\[ I_{es} = \int f_{es}(x) dS, \quad f_{es} = 2a_0 \frac{1+(1+\epsilon)b_0}{1+\epsilon} Re^b u_4(x) \]

It is readily seen that in the case when the ratio \( u_4(x)/u_4(x) \) is constant along the surface of the probe \( I_{es} = I_C \). Otherwise, \( I_{es} > I_C \). Below, we shall to be specific assume that \( I_{es} > I_C \); the case \( I_{es} = I_C \) can be treated similarly.

The complete region \( I > I_{es} \) in which we are interested can be divided into the two regions \( I > I_{es} \) and \( I < I < I_{es} \). It is obvious that for \( I > I_{es} \) (we assume everywhere that \( I-I_{es}=O(Re^{-1/2}) \), \( I-I_{es}=O(Re^{-1/2}) \)) the thickness of the Debye layer has the order \( Re^{-1/2} \) on at least part of the surface of the probe. It is easy to show that the voltage drop across the Debye layer then has the order of magnitude \( \epsilon Re^{-1/2} \), and the voltage drops in the region of the inviscid flow and the quasineutral part of the gas-dynamic boundary layer have as before the order of magnitude \( Re^{1/2} \) and \( \ln \omega^{-1} \). Thus, for \( I > I_{es} \) the main contributions to the total probe–plasma potential difference are made by the region of the inviscid flow and by the Debye layer; the relationship between these contributions is determined by the parameter \( \epsilon Re^{-2} \).

We consider first the case \( \epsilon Re^{-2} \approx 0 \). In this case, the main contribution to the total probe–plasma potential difference when \( I > I_{es} \) is made by the Debye layer, the thickness of this layer on the complete surface of the probe having the order \( Re^{-1/2} \). When \( I_C < I < I_{es} \), we can expect the following in this case. On a certain part \( \Sigma_1 \) of the surface of the probe the Debye layer is homogeneous and asymptotically thin; on the remaining part \( \Sigma_2 \), it is inhomogeneous and asymptotically thin. The thickness of the inhomogeneous Debye layer and the voltage drop across it have the orders of magnitude \( \epsilon^{1/3} Re^{-1/6} \) and \( Re^{-1/2} \). The distribution of the current density on \( \Sigma_2 \) is described by the function \( j_{es}(x) \), and on \( \Sigma_1 \) it can be found by solving the boundary-value problem (3.5)-(3.6) that follows for the distribution of the potential in the region of the inviscid flow:

\[ [j(x)]_{1} = -\left( \frac{\partial \psi}{\partial y} \right)_{1}, \quad \psi = 0 \]  
\[ \nabla^2 \psi = 0 \]  
\[ (\psi)_{1} = \psi_{w}, \quad (\partial \psi/\partial y)_{2} = -j_{es}(x); \quad |r| \rightarrow \infty, \quad \psi \rightarrow 0 \]

At the same time, for given \( \psi_{w} \) the separation of the parts \( \Sigma_1 \) and \( \Sigma_2 \) must be made in such a way that the solution to this problem satisfies the conditions

\[ -\left( \frac{\partial \psi}{\partial y} \right)_{1} < j_{es}(x), \quad (\psi)_{1} < \psi_{w} \]

When the current is increased from \( I_C \) to \( I_{es} \), the fraction \( \Sigma_2 \) of the probe surface increases from 0 to 100%; a further increase in the current is accompanied by an increase in the thickness of the Debye layer over the complete surface of the probe.

It follows from what we have said above that in the case \( \epsilon Re^{-2} \approx 0 \) the order of magnitude of \( dI/\psi_{w} \), which characterizes the slope of the current–voltage characteristic relative to the potential axis, is equal to unity when \( I < I_{es} \) and appreciably less when \( I > I_{es} \) and equal to \( \epsilon Re^{-1/2} \). This change in the slope of the current–voltage characteristic can be called electron saturation.

It is interesting to estimate the value of the potential \( \psi_{es} \) at which \( I = I_{es} \), i.e., saturation occurs. Note that for \( \psi_{w} = \psi_{es}, \quad \psi_{w} = O(Re^{-1}) \), the distribution of the potential in the region of the inviscid flow does not depend on \( \psi_{w} \) in the first approximation and is determined by solving the following exterior Neumann problem:

\[ \nabla^2 \psi = 0, \quad (\partial \psi/\partial y)_{w} = -j_{es}; \quad |r| \rightarrow \infty, \quad \psi_{w} \rightarrow 0 \]

Assuming that \( \psi_{es} \) corresponds to the smallest value of \( \psi_{w} \) at which the voltage drop across the Debye layer \( (\psi_{w} - \psi_{es}) \) is non-negative over the complete surface of the probe, we find \( \psi_{es} = \max \psi_{w}(x) \).

The form of the current–voltage characteristic expected in the case \( \epsilon Re^{-2} \rightarrow \infty \) is shown schematically in Fig. 1 (curve 1).
We now consider the case $\varepsilon \operatorname{Re}^2 \to \infty$. In this case, the main contribution to the total probe—plasma potential difference for $I > I_{es}$ (and, therefore, for $I_c < I < I_{es}$) is made by the region of inviscid flow. In the complete range of currents $I = O(\operatorname{Re}^{1/2})$ the current—voltage characteristic is nearly straight (line 2 in Fig. 1) and in the first approximation is described by (3.3). The distribution of the current density from the probe surface has the form

$$j(x) = -q_w u(x)$$

Finally, for $\varepsilon \operatorname{Re}^2 = O(1)$ the current—voltage characteristic has an intermediate form (curve 3 in Fig. 1). For $I > I_c$ in the general case the voltage drop across the Debye layer on part of the surface of the probe becomes comparable with the voltage drop in the region of the inviscid flow, with the consequence that the characteristic deviates from a straight line. At the same time, the voltage difference across the Debye layer for $I = O(\operatorname{Re}^{1/2})$ is not dominant, and therefore there is no saturation.

Therefore, the conclusion drawn in [7] on the basis of asymptotic analysis of a model with the single small parameter $\varepsilon$ to the effect that there exist saturation currents also applies to the present model, in which not only $\varepsilon$ but also $\operatorname{Re}^{-1}$ is an important small parameter (provided $\varepsilon \operatorname{Re}^2 \to 0$). We note that in [9] a similar condition is obtained for a model in which $\varepsilon$ and $\chi$ are the main small parameters.

The form of the current—voltage characteristics was verified experimentally in an arrangement in which the plasma source was a plasmotron with the following characteristic parameters [10]: power 300 kW, air flow rate 30 g/sec, pressure, temperature, and jet velocity 1 atm, 2500°K, and 300 m/sec, respectively, and weight concentration 0–1% of the additive (potassium atoms). A water-cooled copper probe in the form of a cylinder with a hemispherical top (of radius 0.5 cm) was used; the electrode was the nozzle of the plasmotron forechamber. The ratio of the probe diameter to the dimension of the plasma jet was 0.2, the Reynolds number of the oncoming flow was $\operatorname{Re} \sim 3000$, and $\varepsilon \leq 10^{-7}$.

To measure the current—voltage characteristics, we used the electric circuit shown in Fig. 2 (1 is the probe, 2 the electrode). The scheme allows both dc and ac measurements. For ac measurements, we used an oscillograph, sending to its input the probe—electrode potential difference (abscissa) and a signal proportional to the probe current taken from a shunt in the probe circuit (ordinate).

Mostly, ac measurements were made, since, first, the plasma parameters in the given experimental arrangement fluctuate, so that a detecting effect is manifested because of the nonlinear nature of the electric resistance of the probe; second, there is a flux of solid charged particles formed by erosion of the electrodes of the plasmotron which adhere to the probe, which, probably, explains the appearance of negative pulses perturbing the probe potential and the current to the probe (which were observed by means of the oscillograph). Therefore, dc instruments could introduce an error, measuring certain averaged values of the voltages and currents.
Typical experimental current—voltage characteristics are shown in Fig. 3. We note that in the adopted scale the cathode branch coincides with the negative part of the potential axis. The form of the curves is in complete agreement with the prediction for the given conditions by the theory. Curves 1 and 2 in Fig. 3 correspond to different areas of the collecting surface of the probe (~5 and ~8 cm²), and curves 1 and 3 to different concentrations of the additive (~0.1 and ~1%). The curves were measured with an amplitude 30 V of the alternating voltage. The voltage was limited by the conditions of breakdown in the cathode regime, which was accompanied by a sharp increase in the current. In the anode regime, the characteristic remained linear, and a section of electron saturation was not observed in the investigated range of potentials of the probe (up to 80 V). It is important to note that the probe must be cold the whole time. In our experiment, the probe after long use acquired a film of the additive, which resulted in the appearance of appreciable currents to the probe in the cathode region (the cathode branch of the current—voltage characteristic deviated from the horizontal).

4. Diagnostic Method

We write the expression (3.3) in dimensional variables:

\[ F = 4 \pi \varepsilon \sigma_0 q_o \]  

(4.1)

The capacitance of the probe in this expression depends only on its geometry and is either known from theory or is determined experimentally. In particular, for some of the simplest configurations the theoretical results of [11] can be used.

Given the slope of the experimentally measured current—voltage characteristic, the expression (4.1) permits direct determination of the conductivity \( \sigma_0 \) of the undisturbed plasma. Note that by virtue of the assumptions made in the analysis of the boundary layer this expression is valid, strictly speaking, only for axisymmetric probes with symmetry axis parallel to the oncoming flow. However, since the main contribution to the total probe—plasma potential difference in the region of applicability of the expression (4.1) is made by the region of the inviscid flow, it is to be expected that this expression will also remain valid when the flow around the probe is three dimensional. For the same reason, it can be expected that this expression remains valid when the mean free path of the charged particles is comparable with the thickness of the Debye layer or greater than it.

It is important to emphasize that for the determination of the conductivity of the undisturbed plasma in accordance with the expression (4.1) it is not necessary to know the exact values of the plasma transport coefficients and the rates of the ionization and recombination reactions; it is sufficient to estimate the orders of magnitude of Re, \( \chi \), \( \varepsilon \). If necessary, after \( \sigma_0 \) has been determined one can estimate the concentration \( n_0 x_0 \) of the charged particles in the undisturbed plasma, for which, of course, it is necessary to know the corresponding transport coefficients.
In the case $\varepsilon Re^2 \ll 1$, the values of $\sigma_{\infty}$ and $n_{\infty}x_{\infty}$ can also be found from the values of the saturation currents; such an approach was considered in [12].

Finally, we consider the finding of the potential of the undisturbed plasma from the measured current–voltage characteristic. It is readily seen that to terms of order $kT/e + kT/e$ the potential is equal to the floating potential.

We note that the method of finding $\sigma_{\infty}$ based on the expression (4.1) is valid under the following conditions: the distance between the probe and the corresponding electrode greatly exceeds the dimensions of the probe and the surface area of the electrode appreciably exceeds the surface area of the probe. Under these conditions, the potential of the probe is to be measured from that of the electrode. When it is difficult to satisfy the second condition and the voltage drop across the Debye layer on the electrode can cause the current–voltage characteristic to deviate from the form discussed above, one can introduce into the plasma an additional (reference) electrode under the floating potential and measure the potential of the probe from this electrode. The distance from the probe to the reference electrode must be appreciably greater than the probe dimensions.

The estimate made in the experiment described above of the conductivity on the basis of the slope of the linear section (without a calibration of the readings to take into account the precise geometry of the problem and the inhomogeneous distribution of the plasma parameters) gave the expected order of magnitude for the given varied experimental conditions (0.3–3 S/m). In all cases, the floating potential was equal to the potential of the frame of the plasmotron to within the accuracy of the measurements. Since it can be expected that in the given arrangement the potential of the plasma is close to that of the frame of the plasmotron, this is in complete agreement with the theory developed above.

In conclusion, we note that a method similar to that used above has already been employed in experiments to determine the conductivity of a plasma; we are referring to the electrode method, the essence of which consists of measuring the current in a circuit between two electrodes situated in the plasma and the voltage drop across the interelectrode gap [13]. When this method is used, there are difficulties associated with allowance for the voltage drop in the layers near the electrodes, and also the uncertainty in the discharge configuration. It is important to emphasize that in the framework of the probe method developed here these difficulties do not arise.

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LITERATURE CITED

ON THE DYNAMICAL EQUATIONS FOR LIQUID JETS

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Quasi-one-dimensional equations for the three-dimensional motion of thin liquid jets have been derived by Entov and the present author [1, 2] from the balance integral equations for the mass, momentum, and angular momentum written down for a jet section. Simplified equations of this kind make it possible, in particular, to investigate with comparative ease the motion of bending jets and also the loss of stability of jets moving in air associated with the development of kinks, etc. It is of interest to obtain quasi-one-dimensional equations of jet motion by direct integration over the section of a thin jet of the three-dimensional differential equations of hydrodynamics. In the present note, this approach is illustrated by the example of bending of a jet in a plane.

We shall assume that the section of the jet remains circular when the jet bends. This assumption is valid for sufficiently small amplitudes \( A \) of the bending perturbations of the jet axis of order \((2-4)\alpha_0\), where \( \alpha_0 \) is the initial radius. Indeed, the geometrical distortions of the transverse section at such amplitudes are small, since \( A \) is much less than the wavelength of the perturbation in the considered long-wavelength approximation. On the other hand, energy estimates show that the distortion of the transverse section of a bending jet of sufficiently viscous liquid moving in air by the pressure difference is also unimportant up to \( A \sim (2-4)\alpha_0 \). At larger amplitudes, the rate of distortion of the transverse section increases rapidly and the jet breaks. Thus, until the jet breaks its transverse section maintains its initial circular shape in a bending loss of stability.

The radius vector \( r_a \) of the side surface of the jet is represented by the expression

\[
r_a = R(s, t) + a(s, t)(n \cos \varphi + b \sin \varphi)
\]

where \( a \) is the radius of the jet transverse section, \( \varphi \) is the polar angle in the section, \( R \) is the radius vector of the jet axis, \( n \) and \( b \) are the normal and binormal to the axis, \( s \) is an arbitrary parameter of the axis, and \( t \) is the time.

Accordingly, for the velocity \( v_a \) of a fluid particle on the side surface we have with allowance for the kinematic relation (1.4) of [2]

\[
\frac{dv_a}{dt} = \frac{\partial v_a}{\partial t} + \frac{\partial v_a}{\partial s} \frac{ds}{dt} + \frac{\partial v_a}{\partial s} \frac{\partial s}{\partial t} = U + \frac{\partial a}{\partial t} (n \cos \varphi + b \sin \varphi) + a \left( \frac{\partial n}{\partial t} + \frac{\partial \varphi}{\partial t} \frac{\partial b}{\partial \varphi} \right)
\]

\[
\frac{\partial a}{\partial s} = \lambda \times \frac{\partial a}{\partial t} (n \cos \varphi + b \sin \varphi) + a \left( \frac{\partial n}{\partial s} + \frac{\partial \varphi}{\partial s} \frac{\partial b}{\partial \varphi} \right),
\]

\[
\frac{ds}{dt} = \frac{V - U_t}{\lambda}, \quad \frac{dR}{dt} = \frac{\partial R}{\partial t}, \quad U = \frac{\partial a}{\partial s}, \quad \lambda = \left| \frac{\partial R}{\partial s} \right|
\]

Here, \( V \) is the velocity of a fluid particle on the jet axis, \( U \) is the velocity of a fixed point of the jet axis (the velocity of following), and \( \tau \) is the tangent to the jet axis. The projections of vectors onto the unit axes \( n, b, \) and \( \tau \) are denoted by corresponding indices.