

REVIEW

THEORY OF ELECTRICAL PROBES IN FLOWS OF HIGH-PRESSURE
WEAKLY IONIZED PLASMA

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This review is devoted to the present status of the continuum theory of electrical probes in a moving, weakly ionized plasma. The system of hydrodynamic equations and boundary conditions, determining the distribution of the charged-particle density and potential around a probe, is studied. A system of evaluations which permits analyzing for typical conditions the character of the solution in the region near the probe and the form of the IVC is presented. Works devoted to the calculation of different sections of the IVC are studied. Some questions concerning comparison of theory and experiment as well as possible diagnostic methods are discussed.

1. Introduction. Electrical probes for diagnostics of low-temperature plasma have a number of advantages, including simplicity of the experimental implementation and the possibility of measuring local parameters. Unfortunately the theory of electrical probes in the continuum plasma is quite complicated. The status of this theory in the mid-1970s is presented in the reviews [1, 2]. The purpose of this work is to supplement these reviews and to examine the present status of this theory for the case of a moving, weakly ionized plasma. The analysis is limited to the basic problems in the theory of single stationary probes; more subtle questions, such as double probes [3], probes in nonstationary regimes [4-6], probes in a turbulent plasma [7], and probes in a plasma with an applied electric field [8], fall outside the scope of this work.

2. Determining Equations and Boundary Conditions. We shall study a conducting body (electrical probe) in a moving plasma containing ions (generally speaking, ions of several different types), electrons, and neutral particles (in the general case, also of different types). We shall assume that the degree of dissociation and ionization of the main neutral components (i.e., the components whose concentration is not too low and whose contribution to collisions with charged particles is significant) are low and their molar fractions are virtually constant near the probe. We shall neglect the effect of ionization on the flow field of the neutral components and the effect of collisions between charged particles (with the possible exception of interelectronic collisions) on transport processes. The system of determining equations includes the equation of conservation of charged particles and Poisson's equation [1]

$$n\mathbf{v} \cdot \nabla(n_m/n) + \nabla \cdot \mathbf{J}_m = \dot{n}_m \quad (m=i, e); \quad \Delta\varphi = -4\pi e \sum_{m=i, e} z_m n_m \quad (1)$$

as well as the transport equations, which can be written in the form (see, for example, [9])

$$\mathbf{J}_m = -\mu_m (\nabla p_m / e + z_m n_m \nabla \varphi) \quad (m=i, e). \quad (2)$$

Here n_i , n_e , J_i , and J_e are the densities and diffusion-flux densities of the ions and electrons (the index i runs through the values corresponding to different types of ions); φ is the electrostatic potential; n and \mathbf{v} is the total density and the mean mass velocity of the plasma (this quantity as well as the temperature of the neutrals T can be found by solving the corresponding problem of flow around the probe without taking into account the presence of ionization and here the functions of the coordinates are assumed to be given); \dot{n}_m is the rate of change of the density of the m -th component as a result of volume reactions; e is the electron charge; μ_m , z_m , and p_m are the mobility, charge number, and partial pressure of the m -th component ($p_i = n_i kT$, where k is Boltzmann's constant; the relation between p_e and n_e will be given below). In writing down the transport equations (2) the terms taking into account thermal diffusion and some terms taking into account pressure diffusion have been

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dropped; these effects are usually small and they need be taken into account only in special cases [1, 10, 11].

The hydrodynamic transport equations (2) for ions ($m = i$) are applicable for $\lambda \ll L$, where L is the local value of the mean-free path length of charged particles with respect to elastic collisions (for convenience it is assumed that the mean-free path lengths of the electrons and ions are of the same order of magnitude), and L is the local macroscopic scale, i.e., the characteristic size of the hydrodynamic zone studied.

In the case of a weak field ($eE\lambda \ll kT$, where E is the intensity of the electric field) these equations can be regarded as a particular case of the Stefan-Maxwell relations for multicomponent diffusion [12, 13]; see also [14]. If among the neutral components of the plasma one component is the chief component, then μ_{in} is a function of the temperature only and, to a first approximation [10, 13], equals $3e/(16m_{iq}\Omega_{iq}^{(1,1)})$, where $\Omega_{iq}^{(1,1)}$ is the collisions integral [13, 15] and m_{iq} is the reduced mass; this quantity can be determined either from the experimental data or from results of the calculation of the collision integral (this calculation can be performed based on the known transport cross section or the ion-neutral interaction potential). If the number of chief neutral components of the plasma is greater than 1, μ_{in} also depends on their molar fractions x_q ; to a first approximation, this dependence is described by Blank's law [10] and the mobility is expressed in terms of the mobility in the pure gases μ_{iq}

$$\mu_i = \left(\sum_q \frac{1}{\mu_{iq}} \right)^{-1}, \quad \mu_{iq} = \frac{3e}{16n x_q m_{iq} \Omega_{iq}^{(1,1)}}.$$

In the case of a strong field ($eE\lambda \geq kT$) the diffusion term in the transport equations for ions is of the order of λ/L relative to the drift term and is small when the hydrodynamic approximation is applicable. We note that this is why in most problems studied Einstein's relation with the temperature of the neutrals, which is not applicable in strong fields, can be used to write down the diffusion term in Eqs. (2). The quantity μ_{in} in the case of a strong field becomes a function of E/n also, and Blank's law, generally speaking, is no longer valid. To determine this quantity experimental data or the results from the solution of the kinetic equation must be employed.

An extensive collection of data on the mobilities of ions in pure gases is given in [16, 17].

The hydrodynamic transport equation (2) for electrons ($m = e$) is applicable, if at least one of the following two conditions holds:

$$\lambda_u \ll L, \quad \lambda(v_{en}/v_{ee})^{1/2} \ll L. \quad (3)$$

Here $\lambda_u = \lambda/\sqrt{\delta}$, where δ is a parameter characterizing the transfer of energy from electrons to heavy particles (for elastic collisions it equals twice the ratio of the electron mass to the mass of a heavy particle, while for inelastic collisions it equals the same value multiplied by the coefficient of inelastic losses); v_{en} and v_{ee} are the local values of the electron-neutral and electron-electron elastic collisions frequencies. We note that the scales λ_u , $\lambda(v_{en}/v_{ee})^{1/2}$ are the energy relaxation length of the electrons owing to collisions with neutrals and the maxwellization length owing to interelectronic collisions. The first of these conditions is discussed in [8], while the second condition follows from [19, 20]. When the transport cross sections for electron-neutral collisions are known and the electron distribution function, more precisely its chief - isotropic - part f^0 , is known the mobility of the electrons is determined by Lorentz's formula for a weakly ionized plasma [9, 18, 21].

If the first condition (3) holds, then the function f^0 (and, therefore, the quantity μ_{en}) is determined by the local values of the parameters T , E/n , x_q , and x_e ; in writing down the diffusion term in the transport equation it may be assumed that $p_e = n_e kT$. In the case $E \ll kT/(e\lambda_u)$ f^0 is Maxwell's function with the neutrals temperature and μ_e no longer depends on E/n and x_e . In the case $E \geq kT/(e\lambda_u)$ f^0 for $v_{ee}/v_{en} \leq \delta$ is not, generally speaking, Maxwellian [9, 18, 21]; for $v_{ee}/v_{en} \gg \delta$ f^0 is the Maxwellian function with the temperature T_e , determined by the local balance of Joule heating and energy exchange with neutrals. It is important to note that in the case $E \geq kT/(e\lambda_u)$ the diffusion term in the transport equation for the electrons is of the order of $\lambda_u/L \ll 1$ relative to the drift term, which is what justifies in most problems studied the use of Einstein's relation with the neutrals temperature.

If only the second condition of (3) holds (which is possible, if $v_{ee}/v_{en} \gg \delta$), f^0 is Maxwell function with the temperature T_e . To determine this temperature the system (1), (2) must be supplemented with a differential equation for the electronic energy [1, 9]. In writing down the diffusion term for the transport equation it must be assumed that $p_e = n_e k T_e$.

An extensive collection of data on the mobilities and transport scattering cross sections for electrons is presented in [22-25].

The boundary conditions for the charged-particle density and the potential on the surface of the probe, which is assumed to be ideally absorbing and nonemitting, and far from the probe have the form [1]

$$n_m=0 \quad (m=i, e); \quad \varphi=\varphi_w, \quad (4)$$

$$n_m=n_{m\infty} \quad (m=i, e); \quad \varphi=0, \quad (5)$$

where φ_w and $n_{m\infty}$ are the potential at the surface of the probe relative to the plasma and the charged-particle density in the undisturbed plasma (fixed quantities).

We shall examine briefly the justification for the first boundary condition (4) for ions first. The most systematic derivation is based on the asymptotic analysis of Boltzmann's equation [26]. On the basis of this approach the region of plasma near the wall is divided into a Knudsen layer, i.e., a region of thickness of the order of λ near the wall, and the hydrodynamic region adjacent to it with characteristic linear size $L\delta \gg \lambda$. An asymptotic expansion of the distribution function in the small parameter $\lambda/L\delta$ is constructed in each zone. Joining these expansions gives, in particular, the condition which the first term of the asymptotic expansion, valid in the hydrodynamic region, must satisfy. This condition is the macroscopic boundary condition sought.

The justification of the first condition (4) for the case when the field in the region near the wall is weak is given in [26], and can be formulated as follows. Let n_i^k and $n_i\delta$ be the scales for the ion density in the Knudsen layer and the hydrodynamic region. The flux of ions to the surface in the Knudsen layer can be evaluated, taking into account the anisotropy of the distribution function, in order of magnitude as $n_i^k C_i$, and in the hydrodynamic region as $D_i n_i \delta / L\delta$, where C_i and D_i are the characteristic thermal velocity and coefficient of diffusion of ions. Since across the Knudsen layer the flow is conserved (to within the ratio of the ionization, recombination, attachment, etc. frequencies to the frequency of elastic collisions), these estimates can be equated, whence $n_i^k / n_i \delta \sim \lambda / L\delta \ll 1$. For joining to be possible, the first term of the asymptotic expansion of the distribution function in the hydrodynamic region must vanish on the surface of the probe, whence follows (4).

In the case of a strong field the diffusion term in the transport equations for the ions in the hydrodynamic region, as indicated above, is small compared with the drift term and can be dropped. The order of the degenerate system of hydrodynamic equations obtained in this manner and therefore the number of boundary conditions required for it are reduced. It can be expected that on the basis of this formulation of the problem the boundary condition at the surface is superfluous for ions drifting toward the surface; the first condition of (4) holds for ions drifting away from the surface. On the other hand, the solution that can be obtained on the basis of this formulation equals to order $\lambda/L\delta$ the solution described by the starting formulation, taking into account the diffusion term and using the first condition of (4). Only the density distribution of ions drifting to the surface in the region of thickness of the order of λ near the wall is an exception: on the basis of the degenerate problem this concentration is constant to a first approximation, whereas on the basis of the problem with diffusion it drops to zero (we note, by the way, that both solutions are unphysical in the indicated region and in order to find the true density distribution the first term of the asymptotic expansion of the distribution function in the Knudsen layer must be found). For this reason, in order to give a unified description, which is important, for example, for numerical calculations, Eq. (2) with the diffusion term and the first boundary condition of (4) can be employed also in the case of a strong field; one should only keep in mind that the diffusion drop, obtained on the basis of this approach, in the density of ions drifting toward the surface is unphysical.

The structure of the nonhydrodynamic region near the wall is more complicated for electrons [19, 20]: Aside from a Knudsen layer with thickness of the order of λ , in which the distribution function is anisotropic, there exists another kinetic layer, in which the distribution function is isotropic to a first approximation, but is nonlocal and is not Maxwellian; to determine it the kinetic equation with spatial derivatives must be solved [9, 18]. The thickness of this kinetic layer is much greater than λ and is determined [20] by the smaller of the scales λ_u , $\lambda(v_{en}/v_{ee}^{kin})^{1/2}$, where v_{ee}^{kin} is the electron-electron collision frequency, evaluated from the characteristic electron density in the kinetic layer n_e^{kin} .

If $\delta v_{en} \geq v_{ee}^{kin}$, the thickness of the kinetic layer is of the order of λ_u . In the adjacent hydrodynamic region the first condition (3) holds. In the case $E \ll kT/(e\lambda_u)$, evaluating the electron flux in the kinetic layer and the hydrodynamic region as $n_e^{kin} C_e \sqrt{\delta}$ and $D_e n_e g/Lg$ ($n_e g$, C_e , D_e are the scale of the electron density in the hydrodynamic region, the thermal velocity, and the coefficient of diffusion of electrons) and equating these estimates (to within the ratio of the ionization frequencies, etc., to δv_{en}), we obtain $n_e^{kin}/n_e g \sim \lambda_u/Lg \ll 1$ and the boundary condition (4) is valid. As in the case of a strong field for ions, this condition can also hold for $E \geq kT/(e\lambda_u)$.

If $\delta v_{en} \ll v_{ee}^{kin}$, the thickness of the kinetic layer is of the order of $\lambda(v_{en}/v_{ee}^{kin})^{1/2}$. In the adjacent hydrodynamic region only the second condition of (3) holds, since the quantity λ_u in this case is itself a macroscopic scale - at distances of this order the transfer of electronic energy by heat conduction is comparable to energy exchange with neutrals. Estimating the fluxes in the kinetic layer and the hydrodynamic region as $n_e^{kin} C_e (v_{ee}^{kin}/v_{en})^{1/2}$ and $D_e n_e g/Lg$, we obtain $n_e^{kin}/n_e g \sim \lambda(v_{en}/v_{ee}^{kin})^{1/2}/Lg \ll 1$ and (4) is once again valid.

Thus, the boundary condition (4) is justified for electrons also. We note that in the case $\delta v_{en} \ll v_{ee}^{kin}$, in order to close the hydrodynamic formulation of the problem a boundary condition must be formulated at the surface of the probe for the electron-energy equation. Since this equation [1, 9] has a singularity [1] on the surface of the probe, we shall find its asymptotic solution near the surface. This asymptotic solution includes two terms, one of which is bounded while the other is logarithmic. In this situation the condition that T_e is finite on the surface of the probe can be regarded as the boundary condition sought. We obtain an equivalent form of this equation by writing down on the surface a first-order differential equation which the first of the terms mentioned above satisfies

$$\left(\frac{K_{he}}{kn_e D_e} + \frac{5}{2} \right) k \frac{\partial T_e}{\partial y} = \frac{5}{2} \frac{e \hbar_e}{\mu_e} \left(\frac{\partial n_e}{\partial y} \right)^{-1} - e E_y,$$

where K_{he} is the coefficient of electronic thermal conductivity and the y axis is directed along the normal away from the surface of the probe. This condition can be regarded as a generalization of the conditions obtained by somewhat different methods in [27, 28].

The conditions for applicability of the solution derived on the basis of the foregoing hydrodynamic formulation of the problem are as follows in the general case. The characteristic size of the probe a must satisfy one of the conditions (3). In the case when the solution obtained describes several hydrodynamic zones with different linear scales, one of the conditions (3) must satisfy each of these scales.

3. Solution Methods and Results. With the exception of some very special cases the nonlinear elliptic boundary-value problem (1), (2), (4), and (5) does not have an analytic solution. Numerical solution methods are available primarily for one-dimensional problems [5, 29-34] (as one exception we call attention to [35, 36], where two-dimensional numerical calculations are presented for the case when the charged-particle density in the plasma is low and the Debye radius is comparable to the size of the probe; we also note that in constructing the numerical algorithms the fact that for small Debye radii the Poisson equation is inconvenient for determining φ [34, 37], since the only term in this equation containing φ is small in the main part of the plasma volume, should be taken into account). For this reason approximate approaches, making it possible to calculate one or another section of the IVC, are employed in most works on the theory of probes in flows. We note that to determine the accuracy and range of applicability of these approaches it is natural to use numerical solutions for model one-dimensional problems.

We shall explain the essence of some of these approximate approaches for the often-encountered example of a probe in a plasma flow with a large Reynolds number Re and small Mach number M under conditions when $h \ll \Delta \lesssim d$, where $Re = v_\infty a / \nu_\infty$; ν is the coefficient of kinematic viscosity; h and d are the Debye radius and a scale characterizing volume reactions, for example, the recombination length, evaluated based on the parameters of the undisturbed plasma; $\Delta = a / \sqrt{Re}$ is the scale thickness of the gas-dynamic boundary layer at the probe; the indices ∞ and w here and below are assigned to values corresponding to the undisturbed incident flow and to the surface of the probe. As another example we examine the case of an uncooled probe in a plasma at rest under conditions when $h \ll d \ll a$. In this case everything said below remains valid with the exception that the analogs of the region of nonviscous flow and the gas-dynamic boundary layer will be, respectively, the region of equilibrium ionization and the diffusion layer, $\Delta = d$.

Asymptotic solutions of problems of this type were studied in [38-40]. We shall present some asymptotic estimates, following from these solutions. We shall assume that the first condition of (3) holds in each of the hydrodynamic zones.

The entire region near the probe can be divided into a region of nonviscous flow, the quasineutral part of the gas-dynamic boundary layer, and the volume-charge layer near the wall (the Debye layer or DL). These zones are shown schematically in Fig. 1. In the region of nonviscous flow $n_m = n_{m\infty}$, and in the quasineutral part of the gas-dynamic boundary layer $n_m \sim n_{m\infty}$. The scale of the diffusion flow in all three zones is the same and is determined by the order of magnitude of the diffusion term in Eq. (2) in the quasineutral part of the boundary layer $J_m \sim D_m n_{m\infty} / \Delta$. The electric field in the region of nonviscous flow and in the quasineutral part of the boundary layer is of the order of $kT / (e\Delta)$. In the region of nonviscous flow drift transport of charged particles is much stronger than diffusion transport, while in the quasineutral part of the boundary layer drift transport is comparable to diffusion transport.

To estimate the orders of magnitude of quantities in the DL we shall use the results of the asymptotic analysis, which has been performed many times in the literature for collisional volume-charge layers, starting with [41, 42]; in particular, in [43, 44] the solution is constructed by the method of joined asymptotic expansions in a small parameter (in connection with the work [43] mentioned, we note that the refinements regarding the conditions of applicability of the solution of [42] and the existence of two and not one, like in [41], transitional regions between the main part of the DL and the quasineutral region, made in [43] and cited in the book [1], are not correct [44, 45]). In the case when the DL is uniform (i.e., it does not have an internal structure) [42, 44], diffusion and drift transport in the DL are comparable and are of the order of $D_m n_{m\infty} / \Delta$, whence follow expressions for the orders of magnitude of n_m and E in the layer in terms of its thickness scale y_D : $n_m \sim n_{m\infty} y_D / \Delta$, $E \sim kT / (e y_D)$. Substituting these values into Poisson's equation we obtain $y_D \sim (h^2 \Delta)^{1/3} \ll \Delta$.

In the case when the thickness of the DL is much greater than $(h^2 \Delta)^{1/3}$, but does not exceed Δ in order of magnitude, the layer is nonuniform [41, 43] and includes the main part and the transitional region separating it from the quasineutral region and the surface of the probe, respectively, and the zone of diffusion drop in the density of particles drifting toward the probe. In the transitional region the estimates made above are valid, i.e., $n_m \sim n_{m\infty} (h/\Delta)^{2/3}$, $E \sim kT / [e (h^2 \Delta)^{1/3}]$, and its scale thickness equals $(h^2 \Delta)^{1/3}$ and is much less than y_D . In the main part of the layer the density of particles repelled by the field of the probe is low. For particles that are attracted the main transport mechanism is drift, and their density is of the order of $n_{m\infty} h (y_D \Delta)^{-1/2}$; $E \sim (y_D / \Delta)^{1/2} kT / (eh)$. The thickness of the main part, by virtue of the small thicknesses of the adjacent layers, equals the thickness of the DL as a whole y_D . In the zone of diffusion dropoff the orders of the particle densities are the same as in the main part, drift transport of attracted particles is comparable to diffusion transport, the intensity of the electric field is constant, the scale thickness of this zone equals $h(\Delta / y_D)^{1/2}$ and is much less than both y_D and the scale thickness of the transitional region. Obviously, when the parameter y_D decreases as it becomes comparable to $(h^2 \Delta)^{1/3}$ the structure of the layer degenerates and the estimates presented above become the corresponding estimates for the case of a uniform DL.

It follows from what was said above that the condition for the first inequality of (3) to hold in all hydrodynamic zones is equivalent to the condition $\lambda_u \ll h(\Delta / y_D)^{1/2}$. Obviously, under this condition in all zones f^0 is the Maxwellian function with temperature T .

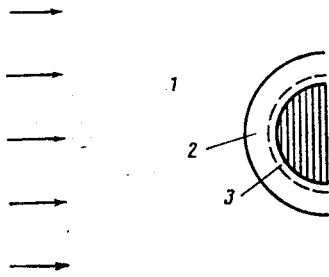


Fig. 1

Fig. 1. Asymptotic structure of the region near the probe: 1) region of nonviscous flow, 2) quasineutral part of the gas-dynamic boundary layer, 3) DL.

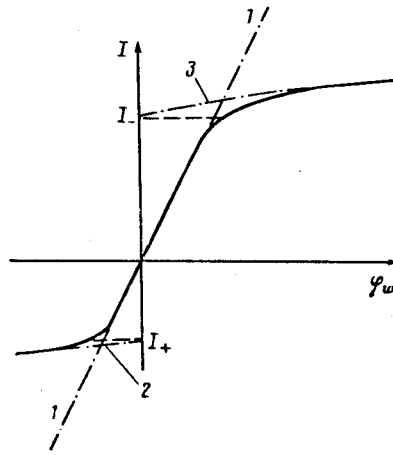


Fig. 2

Fig. 2. IVC of the probe.

In the case of a thin DL, $y_D \ll \Delta$, the generation of particles in the DL by virtue of the condition $d \geq \Delta$ is weak and the fluxes of charged particles toward the probe and, therefore, the current of the probe are determined by the fluxes from the quasineutral region to the outer boundary of the DL. In this connection we shall examine the problem of describing the charged particle density distribution and the electric field distribution in the quasineutral part of the gas-dynamic boundary layer. The system of equations and the boundary conditions on the outer boundary of the boundary layer have the form

$$nv \cdot \nabla (n_m/n) + \partial J_{my} / \partial y = \dot{n}_m; \quad \sum_{m=i,e} z_m n_m = 0, \quad (6)$$

$$J_{my} = \mu_m \left(-\frac{kT}{e} n \frac{\partial}{\partial y} \frac{n_m}{n} + z_m n_m E_y \right), \quad (7)$$

$$y/\Delta \rightarrow \infty, \quad n_m \rightarrow n_{m\infty}. \quad (8)$$

To close this problem boundary conditions at $y = 0$ are required; they are determined from the joining with the asymptotic expansion of the solution in the transitional region (or in the DL as a whole, if it is uniform). Obviously, for the functions n_m this expansion starts with terms of order $n_{m\infty} (h/\Delta)^{2/3} \ll n_{m\infty}$. For this reason, for $y = 0$ the quasineutral densities vanish; we shall write the boundary condition for E_y following [46]:

$$y \rightarrow 0: \quad n_m \rightarrow 0; \quad E_y = \frac{K-1}{K+1} \frac{kT_w}{e} \frac{1}{y} + \dots, \quad (9)$$

where $K = \left(\sum_{m=i,e} J_{my} / \mu_m \right)_w \left(\sum_{m=i,e} J_{my} / \mu_m \right)_v^{-1}$ is a given parameter, characterizing the potential of the probe (the first and second sums extend over components with negative or positive charge, respectively), and all ions are assumed to be singly charged.

It is important to note that in the problem (6)-(9), determining the functions n_m , J_{my} , and E_y in the quasineutral part of the gas-dynamic boundary layer, the effect of the potential of the probe is manifested only through the parameter K . As $K \rightarrow 0$ ($K \rightarrow \infty$), i.e., to suppress the flow of negative (positive) components toward the probe, this effect vanishes and therefore the probe current approaches some constant value, which we shall denote by $I_+(I_-)$.

Thus, in the regime of a thin DL the probe current is bounded by the values I_+ and I_- . We note that $I_+ \sim eD_i n_{i\infty} a^2 / \Delta$, $I_- \sim eD_e n_{e\infty} a^2 / \Delta$.

The foregoing considerations permit analyzing the IVC of the probe [40]. In the range of currents $I_+ < I < I_-$ the DL is uniform, the region of nonviscous flow makes the main contribution to the total potential difference between the probe and the plasma, and the IVC is therefore linear.

In the region $I > I_-$ we have $y_D \sim \Delta$, and the region of nonviscous flow and the DL make the main contributions to the probe-plasma potential difference. These contributions are of order $(kT/e)a/\Delta$ and $(kT/e)\Delta/h$, respectively. The ratio between these contributions is determined by the parameter ah/Δ^2 (we note that this parameter equals, in order of magnitude Reh/a or, which is the same thing, Mh/λ). In the case $ah/\Delta^2 \gg 1$ the contribution of the DL is determining, and the quantity $dI/d\varphi_w$, characterizing the slope of the IVC with respect to the voltage axis, is of the order of $\sigma_\infty a^2 h/\Delta^2$ (σ is the conductivity of the plasma), which is much smaller than the order of magnitude of this quantity in the region $I_+ < I < I_-$, equal to σ_∞ , i.e., the IVC distinctly saturates at $I = I_-$. In the case $ah/\Delta^2 \sim 1$ the contributions are comparable and the IVC has an intermediate form.

In the region $I < I_+$, $y_D \sim \Delta$, the contributions of the region of nonviscous flow and the DL are of the order of $\beta(kT/e)a/\Delta$, $(kT/e)\Delta/h$, where $\beta = D_i/D_e \ll 1$. Depending on the order of magnitude of $\beta ah/\Delta^2$ the IVC is linear or it saturates at $I = I_+$ or it is of an intermediate form.

We note that because the problem is not one-dimensional the limiting current densities on different sections of the surface are not the same and are achieved for different values of φ_w . This effect is not discussed here (see [40]).

Based on the foregoing estimates it may be expected that the potential of the plasma equals, to order kT/e , the floating potential. We note that a more accurate estimate gives for the floating potential of the probe relative to the plasma the value $(kT_w/e) \ln[(h/\Delta)^{2/3} \beta]$; however, the expected accuracy of this estimate is low (logarithmic).

Three relatively simple approaches follow in a natural manner from the foregoing discussion. On the basis of these approaches either the linear section of the IVC of the section of currents of positive or negative particles $I < I_+$, $I > I_-$ (in this case only the voltage drop in the main part of the DL is taken into account) or the saturation currents of the positive or negative particles I_+ , I_- are calculated approximately. The latter two approaches are not applicable for $\beta ah/\Delta^2 \ll 1$ and $ah/\Delta^2 \ll 1$.

Figure 2 shows schematically the IVC of the probe (solid line) and the linear section (1), the sections of the currents of positive or negative particles (2 and 3, respectively), and the saturation currents calculated on the basis of the approaches indicated.

It was assumed above that $\lambda_u \ll h(\Delta/y_D)^{1/2}$, $h \ll \Delta \ll a$. It may be expected that the approaches presented are also valid under less restrictive assumptions. In particular, instead of the first of these conditions it is apparently sufficient to satisfy the inequality $\lambda_u \ll \Delta$. One would expect that in this case the solution obtained on the basis of the studied formulation of the problem will also remain valid in the region of nonviscous flow, the quasineutral part of the boundary layer, and (if $y_D \sim \Delta$) in the main part of the DL, i.e., in the regions that determine the sections of the IVC employed in the approaches described. We note that in calculating the linear section of the IVC, the saturation current of positive ions, and (if $\lambda \ll h$) the section of positive-ion current the effect of the electric field on the transport and kinetic coefficients can usually be neglected (in the region of nonviscous flow and the quasineutral part of the boundary layer f^0 is the Maxwell function with the temperature T , while in the main part of the DL the electron density is low; over the entire volume, including the main part of the DL, the condition $eE\lambda \ll kT$ holds).

The second condition can apparently also be weakened: the first of the approaches enumerated above remains valid also for $h \sim \Delta \ll a$, while the second and third approaches remain valid for $h \ll a \lesssim \Delta$.

We shall first study in greater detail the first approach. The distribution of the potential in the region of nonviscous flow is described by the equation

$$\nabla \cdot (\sigma \nabla \varphi) = 0.$$

Under the assumption $M \ll 1$ in the region of nonviscous flow $\sigma = \sigma_\infty$ (in the case of an uncooled probe in a plasma at rest $\sigma = \sigma_\infty$ in the region of equilibrium ionization because the pressure and temperature are constant), and this equation becomes Laplace's equation.

For this reason the expression describing the linear section of the IVC can be written in the form [40]

$$I/\varphi_w = 4\pi C \sigma_{\infty} \quad (10)$$

The electric capacitance of the probe C , introduced here, depends only on the geometry of the probe. In particular, for a spherical probe with radius R $C = R$; for a probe in the form of a thin rod with length L and radius R $C = L/[2 \ln(L/R)]$; for a diskoid probe with radius R $C = 2R/\pi$; the formulas for a probe in the form of an ellipsoid of revolution are also known (for example, [47, 48]); for a built-in probe in the form of a circle with radius R , placed flush against an immobile plane, $C = R/\pi$; for a probe in the form of a hemisphere with radius R , placed on a nonconducting plane, $C = R/2$, etc. We note that with the help of the results of [49] the formula (10) can be extended to the case of a probe in a magnetic field.

If $M \geq 1$, the conductivity of the plasma in the region of nonviscous flow is, generally speaking, variable and the slope of the linear section is not described by the formula (10). In the case $M \gg 1$ the bow shock wave forming in front of the probe is often set off by a small distance, and the unperturbed region in front of the shock wave may be expected to make the main contribution and (10) remains valid. In the case when the volume reactions at times of the order of the transit time a/v_{∞} are frozen, the molar fractions of the charged particles in the region of nonviscous flow are constant; if, in the process, $\mu_{\infty} n$ is constant for the temperature range studied (this approximation is satisfactory, for example, for a combustion-product plasma), then the conductivity of the plasma in the region of nonviscous flow is constant and (10) is once again valid.

In most works performed on the basis of the second and third approaches, a plasma containing charged particles of two types - electrons and positive ions of one type - is studied, and the ionic section of the IVC or the ionic saturation current are calculated. The sections of the ion current of the IVC of the probes in a plasma with volume ionization and recombination were calculated in [50-52]. In [50] a situation when the generation of charged particles in the DL is weak and the probe current is determined by the diffusion flow of ions to the outer boundary of the DL was studied. In connection with the analysis of probes in a plasma with an external source of ionization or with ionization in collisions with neutral components of the plasma a situation when the contribution of generation to the DL is significant was studied in [51, 52].

The ion-current sections of the IVC of probes in flows of plasma with frozen ionization and recombination for $Re \gg 1$ were calculated in [53, 54]. In addition to these works, in which the case $y_D \sim \Delta$ is studied, there is also a large group of works in which the case $y_D \gg \Delta$ is studied (the so-called "sheath-convection" regime; these works are reviewed in [2], and we also call attention to [55-58]). The distinguished feature of this last case is the simplicity of the calculation of the flow on the outer boundary of the DL, which equals $n_{i\infty} v_{\perp}$ (v_{\perp} is the projection of the mean-mass velocity on the normal to the outer boundary of the DL). To explain this feature we shall examine briefly the buffer zone separating the region of nonviscous flow and the main part of the DL; in so doing we shall assume that $h \ll \Delta \ll y_D \leq a$ and we shall take into account the results of [59]. For $hy_D/\Delta^2 \ll 1$ this zone is analogous to the corresponding zone in the case $y_D \sim \Delta$ and includes the quasineutral diffusion region in which convective, drift, and diffusion transport of charged particles are comparable and the transitional region in which the nonquasineutrality is significant and convective transport is small. Taking into account the fact that $v_{\perp} \sim v_{\infty} y_D/a$, we find the scales of the thickness of these regions: Δ^2/y_D , $(h^2 \Delta^2/y_D)^{1/3}$. For $hy_D/\Delta^2 \sim 1$ these scales are identical; the buffer zone is uniform and all effects enumerated above are important in it. For $hy_D/\Delta^2 \gg 1$ the buffer zone once again becomes nonuniform. It includes a layer in which the electron density drops and the electrons are repelled by the field of the probe, while convection maintains the ion density constant, and a convective-drift layer, in which the ion density decreases from $n_{i\infty}$ to values corresponding to the main part of the DL (it is assumed that $\beta y_D/\Delta^2 \ll 1$). The scale thicknesses of these layers equal h and $h^2 y_D/\Delta^2$.

With the help of the estimates made above it is not difficult to show that the flow of ions across the buffer zone is much stronger than the convective flow of ions in the longitudinal direction. For this reason the flux density of ions across the buffer zone changes little and is determined by the convective and drift flows at distances from this zone much

greater than its thickness. Finding the electric field from the condition that there is no electron flux, we find that this density equals $n_{i\infty} v_1 (1 + \mu_i/\mu_e)$, whence follows, by virtue of the smallness of μ_i/μ_e , the expression presented above.

The voltage drop in the DL for $\Delta \leq y_D \leq a$ is of the order of $(kT/e)y_D^2/(h\Delta)$ (for comparison we note that for $(h^2\Delta)^{1/3} \leq y_D \leq \Delta$ this quantity is of the order of $(kT/e)y_D^{3/2}/(h\Delta^{1/2})$). In order to be able to neglect the voltage drop in the region of nonviscous flow it is necessary that $\beta a h/(\Delta y_D) \ll 1$. If $y_D \ll a$, the main part of the DL can be regarded as locally one-dimensional; if $y_D \sim a$, non-one-dimensional effects must be taken into account.

There are many works on the calculation of the ion saturation current. On the basis of the model of a plasma with two types of charged particles and under the additional assumption that the ratio μ_i/μ_e is constant in the temperature range studied (this assumption holds with adequate accuracy, for example, for a combustion-product plasma) the calculation of the saturation current reduces [38, 60] to the solution of the equation of ambipolar diffusion, for which well-developed methods, including analytical methods, exist.

In the case when volume ionization and recombination are frozen, this equation is linear and the saturation current is proportional to $n_{e\infty}$. Analysis of the results of the calculations [1, 2, 48, 61-64] shows that the ion saturation current toward the probe in the flow with small M is virtually independent of the surface temperature of the probe, and to a first approximation can be calculated neglecting the variability of the transport properties of the plasma. Then the general expression for the ion saturation current toward the probe with a given geometry can be written in the form

$$I_+ = Sh e n_{e\infty} D_{i\infty} a, \quad (11)$$

where Sh is a dimensionless coefficient that depends on Re and Sc :

$$(Sc = v_\infty (D_{i\infty} + D_{e\infty}) / (2D_{i\infty} D_{e\infty}) \approx v_\infty / (2D_{i\infty})).$$

To determine this coefficient it is necessary to find an explicit solution of the corresponding hydrodynamic problem. In particular, for $Re \ll 1$, taking into account two terms in the expansion in terms of Re , the equality [48]

$$Sh = 8\pi \frac{C}{a} \left(1 + \frac{1}{2} Sc Re \right),$$

where $Re = v_\infty C / v_\infty$, holds.

For $Re \gg 1$, $Sh \sim \sqrt{Re}$ and the formula (11) can be written in the form

$$I_+ = F e n_{e\infty} (v_\infty D_{i\infty} a^3)^{1/2},$$

where F is a dimensionless coefficient that depends (usually quite weakly) on Sc .

For a built-in probe on a flat plate this coefficient can be found with the help of the formulas (3.41) and (3.42) from [1]. We note that the analogous formula is given in [2] under the number (31), but the coefficient there is too high by almost a factor of two. The reason for this error is that the asymptotic expansion in the limit $Sc \rightarrow 0$ [65], employed in the derivation of the indicated formula as an approximate solution of the equation of ambipolar diffusion, is inapplicable for real values of Sc , which are of the order of unity.

For a built-in probe on the bow surface of the sphere, F can be found from [63]. We note that a formula for the case when the entire bow part of the surface of the sphere right up to the point of detachment of the laminar boundary layer is of a collection character is given in [2] [formula (34)]. A calculation using this formula gives values that are approximately two times higher than the exact values (obtained numerally) [66]. The saturation current density at the critical point of the cylinder and - on the basis of the analogy between mass and heat transfer and the experimental data on the heat exchange with a cylinder - the saturation current on the full surface of the cylinder were calculated in [67]. If the factor of two lost in [67] owing to the use of an incorrect relation between the saturation current density and the derivative of the quasineutral charged-particle density at the surface of the probe is introduced into the expressions obtained, then the current density at the critical point will be virtually identical to [63, 64], while the total current

will exceed the current calculated based on [63, 64] on the upstream half of the cylinder by ~25%, which gives an estimate of the contribution of the back half for the given conditions [66].

For $Re \sim 1$ the saturation current can be found with the help of the numerical solution of the elliptical problem. Such calculations for the case of a cylindrical probe were performed in [68, 69].

The validity of the above-mentioned formulas of [1] for the wall probe on a flat plate is not limited to the assumption $M \ll 1$. The validity of the formulas (3.43) and (3.44) in [1] for built-in probes on a cone and at the critical point of a blunt body is also not restricted by this assumption [apparently in the formula (3.44) the term $2(1 + \epsilon)$ must be replaced by $\sqrt{2(1 + \epsilon)}$ and $(d\mu/dx)_\delta$ must be replaced by du_δ/dx]. These formulas relate the saturation current with the parameters on the outer boundary of the boundary layer, rather than in the undisturbed flow. The formulas relating the saturation current on a spherical probe to $n_{e\infty}$ in a flow with large M and moderate Re , when the flow around the probe occurs in the regime of a viscous shock layer, were derived in [70, 71]; the results of a numerical calculation of the charged-particle density distribution and the potential distribution on the critical line of a viscous shock layer are presented in [72].

In the case when volume ionization and recombination in the vicinity of the probe are significant, the equation of ambipolar diffusion becomes nonlinear. In the limit of rapid flow of reactions ($d \ll \Delta$) this equation can be solved analytically [73]. This solution, however, relates the saturation current not with $n_{e\infty}$, but rather with some value of the equilibrium density in the perturbed region. For this reason the saturation current can be used to determine $n_{e\infty}$ in this limit only in the case when the equilibrium concentration in the vicinity of the probe is constant.

The saturation current on probes in flows with large Re with nonequilibrium flow of reactions ($d \sim \Delta$) was calculated in [74-78]. In all these works, except for [76], numerical methods were employed; in [76] an approximate expression for the recombination rate that is linear in the charged-particle density was employed.

We note that the saturation currents on strongly cooled and uncooled probes in a plasma with thermal ionization depend substantially differently on $n_{e\infty}$. In the case of a strongly cooled probe the rate of thermal ionization near the probe is low, and recombination has the determining effect on the current. Since the recombination coefficient increases as $n_{e\infty}$ increases, the current grows more slowly than $n_{e\infty}$. In the case of an uncooled probe as $n_{e\infty}$ increases the spatial scale of the change in the charged-particle density at the surface of the probe, proportional to d , decreases, as a result of which the current grows more rapidly than $n_{e\infty}$.

In the works enumerated above we studied a situation when the temperature profile near the probe is monotonic. A formula for the saturation current on a built-in probe on a sharp cone, when the local Mach number of the flow is large and the temperature profile in the boundary layer has a maximum near which ionization occurs, is given in [1] under the number (3.47) and is refined in [60].

In the works enumerated above, the saturation current was calculated taking into account the presence of positive ions of one type and electrons in the plasma. The case when negative ions of one type are also present while reactions are frozen was studied in [79-81]. It is shown in [79, 80] that the presence of negative ions leads to a small increase in the saturation current of positive ions on a built-in probe on a flat plate in a boundary-layer flow. In [81], in an analysis of a probe at the critical point of a blunt body in a subsonic flow of plasma in the boundary-layer regime, the unjustified [1, 82] assumption that the diffusion is ambipolar in the quasineutral region was made. The case when the plasma contains ions of many types and reactions occur was studied in [46] in connection with the calculation of the saturation current on spherical and cylindrical probes in a subsonic boundary-layer flow of a combustion-product plasma seeded with potassium or sodium. On the basis of the model employed the presence of negative ions leads to quite appreciable increase in the positive-ion saturation current.

4. Comparison of Theory with Experiment. Both the quantitative and qualitative comparison of different aspects of the theoretical results mentioned in Sec. 3 with experimental data have been performed many times in the literature (see the reviews [1, 2]; of subsequent

works we call attention to [33, 34 40, 46, 48, 51, 54-57, 62, 64, 66-69, 74, 75, 77, 78, 83-93]). In most cases the agreement between theory and experiment was reasonable. In particular, the quantitative accuracy is of the order of a factor of two. We shall not give a detailed comparison, but we call attention to two points.

In a number of works the theory was compared with the data obtained in experiments with inert-gas plasma. In such a plasma the parameter δ is very small (of the order of 10^{-4}). The quantity λ_u even under atmospheric pressure may not be too small, so that the first condition of (3) often does not hold. On the other hand, the region of parameters in which the second condition of (3) and the condition $v_{ee} \ll v_{en}$ hold simultaneously is often quite narrow. For example, for an argon plasma at atmospheric pressure, $T = 2 \cdot 10^3$ K, $\Delta = 10^{-1}$ cm the mean-free path length of the electrons is of the order of 10^{-2} cm, $\lambda_u \sim 1$ cm, and the condition $\lambda_u \ll \Delta$ does not hold. The condition $(\lambda/\Delta)^2 \ll v_{ee}/v_{en} \ll 1$ holds for $10^{11} \text{ cm}^{-3} \ll n_e \ll 10^{13} \text{ cm}^{-3}$. Thus, in the case $n_e \leq 10^{11} \text{ cm}^{-3}$ the hydrodynamic formulation of the problem under these conditions is, generally speaking, not applicable; in the case $n_e \geq 10^{13} \text{ cm}^{-3}$ the hydrodynamic formulation is possible, but the electron-ion collisions must be taken into account; in the case $n_e \sim 10^{12} \text{ cm}^{-3}$ the formulation of Sec. 2 can be used, but the theoretical results mentioned in Sec. 3, which do not take into account the separation of the electron temperature, are not valid. Works devoted to the theory of probes for these cases have been published. For the first case we call attention to [94]. Some works referring to the second and third cases are cited in [1], and for the second case we also call attention to [95-98]. On the whole, however, the present status of the theory for these conditions is appreciably lower than for a weakly ionized plasma with an equilibrium function f^0 . For this reason, in order for the quantitative comparison with the experimental results for inert-gas plasmas in a wide range of conditions to be methodically correct, the theory must be further developed for the indicated cases.

In a significant number of works the theory was compared with data obtained in experiments with a combustion-product plasma with an alkaline additive. Because of the large values of the transport cross sections and cross sections of inelastic collisions of electrons with H_2O and CO_2 molecules the lengths λ and λ_u in such a plasma are usually short (for example, at atmospheric pressure and $T = 2000$ K: $\lambda \sim 10^{-4}$ cm, $\lambda_u \sim 10^{-3}$ cm) and the conditions for the applicability of the theoretical results mentioned in Sec. 3 hold. On the other hand, the difficulties in performing probe experiments in a high-energy flow of a combustion-product plasma are surmountable [91, 99-101]. In the last few years, however, in connection with measurements of the binding energy of the negative ions HCO_3^- [102], a suspicion has appeared that taking into account these ions could substantially affect the interpretation of the IVC, in particular, the positive-ion saturation current [46]. One must hope that this important question will be clarified in the near future.

5. Conclusion. Diagnostic Methods. Three diagnostic methods, based on the use of the linear section of the IVC or the section of current of positive particles or the values of the saturation current of the positive particles follow from the foregoing discussion. Apparently the first method is the simplest one in many cases. Indeed, the formula (10) permits determining σ_∞ directly from the slope of the linear section of the IVC measured in the experiment. In so doing it is not necessary know the exact values of the transport and kinetic coefficients of the plasma. After σ_∞ is determined $n_{e\infty}$ can be found if necessary, for which, of course, one must know μ_e . We call attention to the fact that this method is related with the electrode method for determining the conductivity of a plasma, which is based on measuring the IVC of the gap between two electrodes placed in a plasma [100, 103, 104]. On the other hand, none of the difficulties, associated with taking into account the voltage drop in the layers near the electrodes and the uncertainty of the configuration of the discharge, that arise in the electrode method appear in the method under study. A scheme enabling the use of such a method for monitoring the conductivity of a plasma directly during the course of an experiment is described in [40] (see also [93]). On the whole, however, this method has not been adequately tested yet.

In the case when the plasma contains ions of only one type while ionization and recombination are frozen, the second and third methods also can be quite easily implemented, often in the form of explicit formulas relating the expression for the section of the current of positive particles or the magnitude of their saturation current to $n_{e\infty}$. We note that the values of the transport coefficients of the plasma, required for determining $n_{e\infty}$ with the help of these formulas, can in many cases be calculated with acceptable accuracy (with an error not exceeding $\sim 10\%$ [16]). For a multicomponent, chemically active plasma

these methods are much more complicated and they require that the kinetics of ionization and recombination in the region near the probe be taken into account. The existing information regarding this question is often inadequate. For this reason in this case these methods are of limited use for determining the charged-particle density in an undisturbed flow. On the other hand, they can be employed to study the indicated kinetics.

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