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Multiple solutions in the theory of dc glow discharges and cathodic part of arc discharges. Application of these solutions to the modeling of cathode spots and patterns: a review

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Abstract
A new class of stationary solutions in the theory of glow discharges and plasma–cathode interaction in ambient-gas arc discharges has been found over the past 15 years. These solutions exist simultaneously with the solution given in textbooks, which describes a discharge mode with a uniform or smooth distribution of current over the cathode surface, and describes modes with various configurations of cathode spots: normal spots on glow cathodes, patterns of multiple spots recently observed on cathodes of glow microdischarges and spots on arc cathodes. In particular, these solutions show that cathode spots represent a manifestation of self-organization caused by basic mechanisms of the near-cathode space-charge sheath; another illustration of the richness of the gas discharge science. As far as arc cathodes are concerned, the new solutions have proved relevant for industrial applications. This work is dedicated to reviewing the multiple solutions obtained to date, their systematization, and analysis of their properties and physical meaning. The treatment is performed in the context of general trends of self-organization in bistable nonlinear dissipative systems, which allows one to consider glow discharges or arc–cathode interaction within a single physically transparent framework without going into mathematical details and offers a possibility of systematic computation of the multiple solutions. Relevant computational aspects and experimental data are discussed.

Keywords: gas discharges, electrodes of gas discharges, electrode spots, patterns of spots, glow discharges, arc cathodes

(Some figures may appear in colour only in the online journal)

1. Introduction

Self-consistent theoretical models of dc glow discharges and plasma–cathode interaction in arc discharges in ambient gas, including the most basic ones, admit multiple solutions existing for the same discharge current. One of these solutions is in the simplest case one-dimensional (1D) and describes states with a uniform distribution of current over the electrode surface. In the case of glow discharges between parallel electrodes, the 1D solution describes the Townsend discharge for very low current densities, the abnormal discharge for high current densities, and the unstable discharge with a falling current density–voltage characteristic (CDVC) for intermediate current densities; this solution is similar to the classic solution which is based on a linear approximation of electric field in the near-cathode space-charge sheath and is given in textbooks (e.g. [1, section 8.4.2] and [2, section 14.3]). In the case of arc–cathode interaction, the 1D solution describes the diffuse mode of current transfer and is similar to the solution which for high-pressure arcs is considered in [3, section 4]. The other existing solutions are in all the cases multidimensional and describe modes with different configurations of cathode spots.

The existence of multiple solutions was hypothesized in 1963 for arc–cathode interaction [4] and derived in 1988 for glow discharges [5]; further references of historical interest can be found in [6, section 3.1] and [7], respectively. However, the central role of multiple solutions was fully realized only in the late 1990s in the theory of arc plasma–cathode interaction. By now, solutions describing diffuse and spot modes of current transfer to cathodes of high-pressure arc discharges have been computed under different conditions by different research groups and validated by an extensive comparison with the experiment. Most of the effort was invested in investigation of low-current high-pressure arcs, which are used in high-intensity discharge lamps. Multiple solutions in the theory of glow discharges have started to be systematically computed and validated experimentally only recently.

Although the physical mechanisms of plasma–cathode interaction in dc glow and arc discharges are very different, the overall patterns of multiple solutions turned out to be remarkably similar. Of course, this is not surprising: in terms of general theoretical physics, cathodic parts of both discharges represent bistable nonlinear dissipative systems; cathode spots represent self-organization phenomena. Hence, solutions describing the spots must conform to general trends of self-organization in bistable systems. This allows one to understand multiple solutions in the theory of glow and arc cathodes and different spot patterns described by these solutions within the same framework.

These solutions are important also beyond their usefulness for understanding and modeling cathode spots in dc discharges. The fact that the model of dc glow discharges between parallel electrodes, which is the workhorse of the gas discharge theory and modeling, admits a new class of multidimensional solutions and these solutions are of physical relevance is by itself surprising and theoretically interesting. These solutions add to understanding of the physics contained even in simple gas discharge equations; another illustration of the richness of the discharge science. Furthermore, understanding of these solutions may also be important in apparently simple situations where multiple solutions are not of primary concern.

The latter point may be illustrated by the following example [8]. In figure 1, current–voltage characteristics (CVCs) are shown of a dc glow discharge between parallel electrodes calculated in the framework of a simple drift–diffusion model. The solid line refers to the case where the lateral surface of the discharge vessel reflects the ions and the electrons, in which the discharge is described by the 1D solution (all parameters vary only in the axial direction). The dashed line refers to the case where the lateral surface absorbs the ions and the electrons, so the solution is axially symmetric (two-dimensional, 2D). The 2D solution is close to the 1D solution in the Townsend and abnormal regimes; however, at intermediate currents it describes the subnormal and normal modes rather than the mode associated with the falling section of the CVC. Note that the only new effect in the 2D solution is diffusion of the charged particles to the (absorbing) wall.

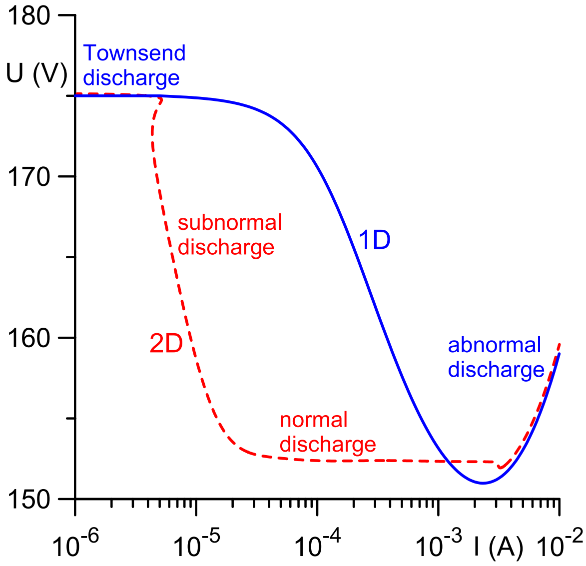

Figure 1. Computed CVCs of the glow discharge. 1D: reflecting lateral surface; 2D: absorbing lateral surface; Xe plasma, $p = 30$ Torr, the discharge radius 1.5 mm and height 0.5 mm. Adapted from [8].

This effect is weak: the ratio of the electron current to the wall to the discharge current, evaluated with the use of the 2D solution, is of the order of 10^{-3} or lower at all discharge currents. Then a question arises as to how this weak effect originates such a large difference, in particular, where from have the subnormal and normal modes appeared and where to has the mode associated with the falling section of the CVC gone. Identification and understanding of multiple solutions are indispensable in answering this question.

This work is dedicated to a review of multiple solutions in the theory of dc glow discharges and plasma–cathode interaction in arc discharges obtained to date, their systematization, and analysis of their properties and physical meaning. The outline of the paper is as follows. In section 2, the concept of multiple solutions in the theory of dc glow discharges and plasma–cathode interaction in arc discharges is formalized and properties of these solutions are analyzed on the basis of general trends of the theory of self-organization in bistable nonlinear dissipative systems. Relevant aspects of computation of these solutions are discussed in section 3. Typical results of calculations of multiple solutions are shown and compared with trends observed in the experiment in section 4. Topics discussed in section 5 include the following: transition from self-organized modes of current transfer to modes where current spots represent concentrations of current caused by non-uniformities of geometrical and/or physical properties of the cathode surface; solitary cathode spots; role of Steenbeck's principle of minimum power in modern theory and modeling; examples of apparently simple situations where glow discharges or arc–cathode interaction reveal complex behavior; observations of spots and patterns on electrodes of gas discharges and the first-principles theory and modeling where available. The place of the approach based on multiple steady-state solutions in the theory and modeling of gas discharges and possible directions of future work are briefly discussed in section 6.

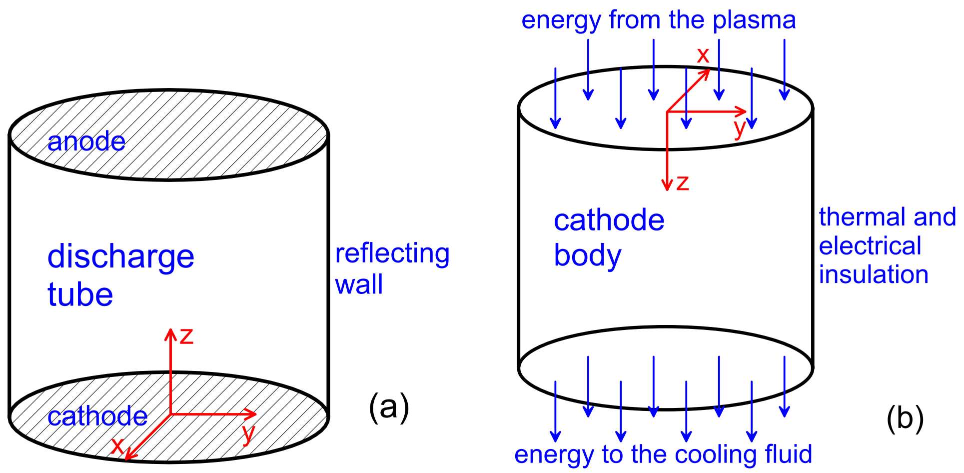

Figure 2. Geometry admitting a 1D solution; (*a*) glow discharge and (*b*) cathode of an arc discharge.

2. Theory of multiple solutions

2.1. Mathematical formulation

The concept of multiple solutions in the theory of dc glow discharges and the cathodic part of arc discharges may be formalized as follows. It is convenient to consider first a discharge with parameters varying in the direction along the electric current but not in transversal directions and then gradually move to configurations of practical interest; it is in this way how the multiple solutions have actually been understood and computed. Appropriate geometry for the case of a glow discharge is shown in figure 2(*a*): a discharge in a tube with parallel electrodes and without recombination of the charged particles at the lateral wall (i.e. with the wall reflecting the charged particles). The z-axis of the Cartesian coordinates (x, y, z) is parallel to the electric current. Differential equations governing glow discharges are well known and comprise equations of conservation and transport of the ions and the electrons, the Poisson equation, and other relevant equations such as equations of conservation and transport of excited neutral species and the electron energy equation. In the configuration being considered, these equations supplemented with the appropriate boundary conditions admit a 1D solution describing states in which plasma parameters vary across the discharge gap but not in the transversal directions: $f = f(z)$. This solution is exemplified by the solid line in figure 1 and is similar to the classic solution given in textbooks; e.g., [1, section 8.4.2] and [2, section 14.3].

In the case of an arc discharge, the computation domain includes not only the plasma but also the electrodes, where distributions of the temperature and, in some cases, electrostatic potential need to be computed. However, this paper is concerned with the cathodic part of the discharge, and this allows us to reduce the computation domain exploiting the fact that a very substantial electric power is deposited by the arc power supply into the near-cathode space-charge sheath, which is why the energy flux to the cathode surface is generated in a very thin near-cathode plasma layer comprising the space-charge sheath and the adjacent ionization layer. The plasma–cathode interaction is, to the first approximation, unaffected by the arc column and governed by the boundary-value problem, which comprises multidimensional differential equations of heat conduction and current continuity in the cathode body coupled through boundary conditions with a system of transcendental equations describing current transfer through the near-cathode layer. This approach is sometimes called the model of nonlinear surface heating and was proposed for the first time apparently in [4]; more recent versions are described in [6] and references therein. Appropriate geometry is shown in figure 2(*b*): the cathode has the shape of a right cylinder; the lateral surface is thermally and electrically insulated, so the energy flux and the electric current from the plasma enter the cathode through the front surface (the upper end of the cylinder in figure 2(*b*)); the base of the cathode is externally cooled. Again, the governing boundary-value problem admits a 1D solution describing states in which the temperature and potential vary in the direction from the front surface to the base of the cathode but not in the transversal directions: $f = f(z)$. This solution is similar to the one considered in [3, section 4].

The above-discussed 1D solution in both cases of glow discharge and arc cathode describes a mode with a uniform current distribution along the cathode surface, i.e. the spotless, or diffuse, mode. The question is: do the same equations in the same geometry and with the same boundary conditions admit also multidimensional solutions, $f = f(x, y, z)$? If they do, will such solutions have physical meaning, in particular, will they describe modes with cathode spots?

Although 2D and three-dimensional (3D) nonlinear boundary-value problems are solved numerically as a matter of routine nowadays, finding multiple solutions is a non-trivial task which can hardly be solved by purely computational means. A rule of thumb says that if iterations in a nonlinear multidimensional problem with multiple solutions converge painlessly, then the converged solution is likely to be not the one being sought. Sufficient qualitative information must be available in advance in order for all relevant multidimensional solutions to be computed; in the first place, one needs to know what the multidimensional solutions are like and where to look for them. (Of course, such information will

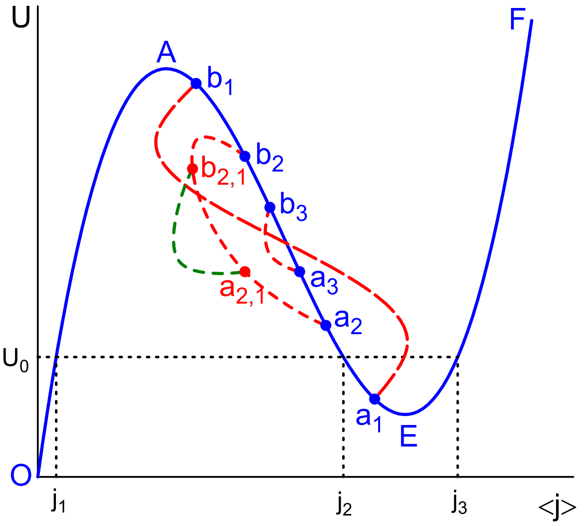

Figure 3. Schematic of CVCs of the glow discharge and the cathodic region of the arc discharge described by different solutions; solid: 1D solution (spotless mode); dashed: multidimensional solutions (spot modes); circles: bifurcation points.

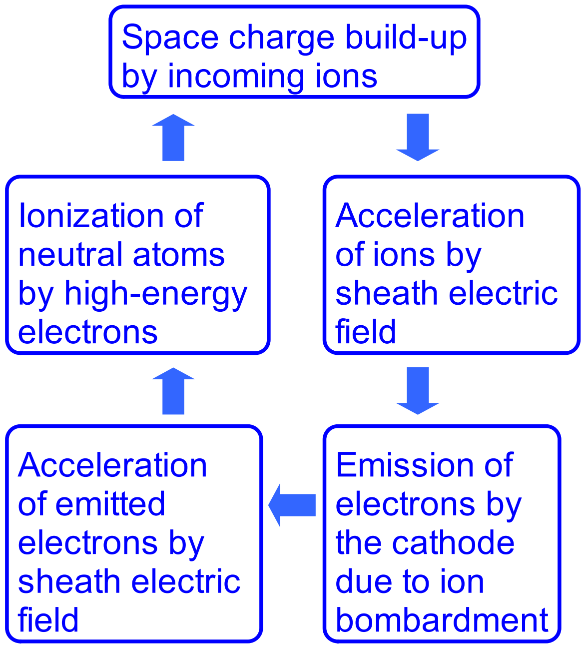

Figure 4. The cathode sheath instability mechanism.

also facilitate analysis of computation results.) This can be achieved by invoking the theory of self-organization in nonlinear dissipative systems. The route from general ideas of the theory of self-organization to computation of multiple solutions describing particular dc discharges is traced in the following sections.

2.2. Deriving properties of multiple solutions from general trends of self-organization in bistable nonlinear dissipative systems

The CVC of the spotless mode of the glow discharge under conditions of figure 1 includes, in addition to the branch depicted by the solid line, a branch that corresponds to no discharge being ignited and represents a section of the voltage axis from zero up to approximately 175 V (the breakdown voltage). This CVC is schematically shown by the solid line $OAEF$ in figure 3. (Here j is the density of electric current from the plasma to the cathode surface and $\langle j \rangle$ is the average density of current to the cathode surface. For brevity, $\langle j \rangle$ will also be referred to as the discharge current.) Note that section OA in figure 3 is moved away from the axis of voltages for illustrative reasons.

It will be seen later that the line $OAEF$ schematically represents the CVC of the spotless mode also in the case of the arc cathode, provided that U, while designating the discharge voltage in the case of the glow discharge, designates the near-cathode voltage drop in the case of the arc cathode. Thus, the line $OAEF$ in figure 3 may be viewed as a prototypical CVC of the spotless mode in the cases of both glow discharge and arc cathode. Since $j = \langle j \rangle$ for the spotless mode, this line also represents the CDVC of the spotless mode.

The CDVC $OAEF$ has a characteristic shape resembling the letter N, which is typical of a bistable system. A characteristic feature of bistable systems is the presence of a strong positive feedback; it is this feedback that gives rise to the falling section of the N-shape (section AE in figure 3). The positive feedback in the considered system originates in the near-cathode space-charge sheath. The relevant mechanism, which may be called the cathode sheath instability, may be described in the same terms for both glow and arc cathodes as illustrated by figure 4. Of course, the emission of electrons by glow (cold) and arc (hot) cathodes is of a different nature: secondary electron emission versus thermionic/thermo-field emission. (One of the consequences is different efficiency of ion bombardment: the number of electrons emitted per incident ion is typically of the order of 10^{-2}–10^{-1} for glow cathodes and of 1–10 for arc cathodes.) Among other things, this difference results in different interpretations of the positive feedback in the mechanism shown in figure 4: in the case of the glow discharge, the positive feedback originates in an increasing dependence of the Townsend ionization coefficient on the electric field, while the positive feedback in the case of the arc cathode originates in an increasing dependence of the density of the energy flux from the plasma to the cathode surface on the surface temperature [9].

Let us denote by U_0 a value of the voltage U somewhere between the values corresponding to the maximum and minimum points of the CDVC: $U_E < U_0 < U_A$. Three 1D solutions exist for $U = U_0$, as shown in figure 3. The states described by the solutions with the lower and higher current densities, $j = j_1$ and $j = j_3$, are expected to be stable and will be referred to as the cold and hot phases, respectively. The state described by the 1D solution with the intermediate current density, $j = j_2$, is expected to be unstable. One can also think of multidimensional solutions existing for $U = U_0$ which would describe states in which a part of the cathode is occupied by the cold phase and the other part by the hot phase; there is also an intermediate region separating the two phases, which is shown in figure 5 and is frequently called the domain wall. Of course, coexistence of phases is

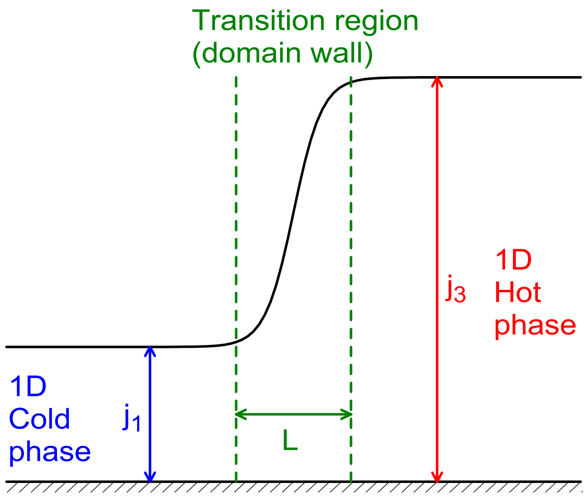

Figure 5. Current density distribution over the cathode surface characteristic for coexistence of phases.

only possible if width W of the cathode, i.e. its characteristic dimension in the directions perpendicular to z, is much larger than the characteristic width L of the domain wall shown in figure 5. (In the case of the glow discharge, one can speak not of the cathode width but rather of the width of the discharge tube, which is equivalent; see figure 2(a).) With a reduction in W, the form of multidimensional solutions deviates from that with the coexistence of phases shown in figure 5, the range of their existence shrinks, and starting from a certain value of W the multidimensional solutions cease to exist.

Switching from the spotless mode to a mode with spot(s) is a result of development of an instability of states belonging to the spotless mode with respect to multidimensional perturbations. The expected pattern of instability is as follows. States corresponding to the rising section EF of the CDVC of the spotless mode (figure 3) are usually stable. As the current decreases into the range corresponding to the falling section AE, multidimensional perturbations of one mode (let us call this mode the first one) start growing. Let us denote the state at which this happens by a_1. The real part of the increment of perturbations of the first mode vanishes at a_1, so the increment is either zero or imaginary. If the former is the case, then the perturbations of 1D states belonging to the beginning of the section a_1A grow in time monotonically. If the latter is the case, then the perturbations grow in an oscillatory way.

It is known from the experiment that the transition from the abnormal discharge on a glow cathode to the normal discharge is monotonic in time; in particular, there are no oscillations of luminosity of the cathode surface. The diffuse-spot transition on arc cathodes is monotonic as well. Hence, it should be expected that the increment of the first-mode perturbations at the state a_1 is zero. In other words, multidimensional perturbations of the first mode are stationary at the state a_1. The latter means that a steady-state multidimensional solution branches off from the 1D solution at this state. In other words, the state a_1 belongs simultaneously to the 1D solution and to another steady-state solution which is multidimensional at states other than a_1. This phenomenon, called bifurcation, or branching, of solutions, is well known in mathematical physics and frequently occurs in nonlinear systems possessing symmetries.

The rising section OA is stable, hence the first-mode perturbations return to being decaying somewhere between the states a_1 and A. Let us denote the state at which this happens by b_1. It seems natural to assume that the steady-state multidimensional solution, which branches off at the state a_1, rejoins the 1D solution at the state b_1, as shown by the dashed line a_1b_1 in figure 3. Note that this reasoning may seem to contradict the well-known experimental fact that the loss of stability of the Townsend discharge may be oscillatory; e.g. [10–17] and references therein. However it does not, since stability in the vicinity of the maximum A of the CDVC and minimum E may be broken by perturbations of different modes. In other words, one should not be surprised that, while all states of the section Ea_1 are stable, some states of the section Ab_1 may be unstable against oscillatory perturbations, as indicated by the above-mentioned experimental fact.

In addition to a_1b_1, other steady-state multidimensional solutions which branch off from and rejoin the 1D solution may exist. The number of such solutions increases with increasing cathode width. Two of these solutions are exemplified by the lines a_2b_2 and a_3b_3 in figure 3. The order of rejoining of different solutions is usually inverse to the order of their branching-off, i.e. the solution a_2b_2 is the second to branch off and the last but one to rejoin, and so on.

Furthermore, there may be multidimensional solutions that branch off from the 1D solution not directly but rather through two or more sequential bifurcations; e.g. $a_{2,1}b_{2,1}$ in figure 3. In other words, one can expect that there are several 'generations' of solutions. The 1D solution represents the first generation. Multidimensional solutions of the second generation (a_1b_1, a_2b_2, a_3b_3 in figure 3) are those that branch off from the 1D solution. Note that the corresponding bifurcations (those occurring at states a_i and b_i) are called primary. Multidimensional solutions of the third generation ($a_{2,1}b_{2,1}$ in figure 3) branch off from solutions of the second generation through secondary bifurcations (those occurring at states $a_{2,1}$ and $b_{2,1}$), etc.

Thus, we come to the conclusion that multidimensional solutions exist if the cathode is sufficiently wide; if they exist, some of them branch off from the 1D solution and rejoin it through bifurcations occurring on the falling section of the CDVC described by the 1D solution; other multidimensional solutions bifurcate from the 1D solution not directly but rather through a chain of sequential bifurcations.

An important particular case is the one where the calculation domain (the glow discharge tube or the body of the arc cathode) represents a right circular cylinder. It is natural in this case to use cylindrical coordinates (r, ϕ, z) with the origin at the center of the cathode surface. Solutions of the second generation are either axially symmetric (2D) or 3D with an arbitrary azimuthal period (2π, or π, or $2\pi/3$, or $\pi/2$, etc). Note that what branches off at a given primary-bifurcation point is not a single 3D solution but rather a continuum of solutions differing by azimuthal orientation of the spot arrangement; however, the term 'solution' will be used for brevity.

Secondary bifurcations can occur on 2D solutions and every other 3D solution of the second generation, i.e. on 3D solutions with one of the periods π, $\pi/2$, $\pi/3$, $\pi/4$, etc. In both cases, the bifurcating third-generation solution is 3D. In the former case, its period may be arbitrary. In the latter case, its period equals double the period of the second-generation solution from which it bifurcates; the so-called period-doubling bifurcation. Note that figure 3 may be viewed as an example of the situation where a third-generation solution with azimuthal period 2π $(a_{2,1}b_{2,1})$ branches off from a second-generation solution with the period π (a_2b_2). Thus, all solutions of the third and subsequent generations are 3D and all tertiary and subsequent bifurcations are period-doubling.

A detailed analysis of bifurcations in the considered problem can be found in [8, 18] for the glow discharge and [8, 9, 19] for arc cathodes.

2.3. Solutions describing regimes with normal spots

An important limiting case is the one where W the width of the cathode (we recall that in the case of a glow discharge it coincides with the width of the discharge tube) is much larger than the characteristic length scale L of the domain wall separating the cold and hot phases. A mathematical theory for this case is developed in [20]. A CVC described by the first multidimensional solution (the one shown in figure 3 by the line a_1b_1) in this case is depicted by the line a_1GHb_1 in figure 6. As L/W becomes smaller, the bifurcation point a_1 comes nearer to the point of minimum E of the CDVC of the spotless mode; the bifurcation point b_1 comes nearer to the point of maximum A; the section HG becomes closer to the horizontal line $U = U_0$; the sections a_1G and b_1H become closer to the sections EM and AK, respectively.

The distribution of the current density over the cathode surface described by the first multidimensional solution evolves in the region of existence of this solution as follows. At the state a_1, this distribution is uniform. In the beginning of the section a_1G, the uniformity is broken and a cold domain surrounded by a hot phase (that is, a small region with the current density lower than that in the surrounding zone) begins forming. As the vicinity of the state G has been reached, the current density in this domain falls down to the value j_1 and the current density in the surrounding region occupied by the hot phase increases to j_3. On the section GH, the region occupied by the cold phase expands and the region occupied by the hot phase shrinks, the current densities in these regions being nearly constant and equal to j_1 and j_3, respectively. These are regimes with coexistence of phases shown in figure 5 or, in other terms, regimes with a normal spot. At the state H the cold phase occupies the whole cathode surface except for a small region (a current spot). This current spot vanishes gradually on the section Hb_1 and the current distribution regains uniformity at the state b_1.

The above-described features characteristic for the case of small L/W are also present in the second and third multidimensional solutions (the ones shown in figure 3 by the lines a_2b_2 and a_3b_3); however, they are successively less pronounced. In particular, the horizontal section of the CVC, which corresponds to the regime of coexistence of phases and for the first solution is represented by the segment HG in figure 6, is successively shorter and more inclined. The reason is clear: higher order multidimensional solutions describe modes with several spots, and as the number of spots increases, the cathode area per spot becomes smaller and the asymptotic features become less pronounced. High-order multidimensional solutions, which describe modes with many spots and are not shown in figure 3, do not manifest these features at all.

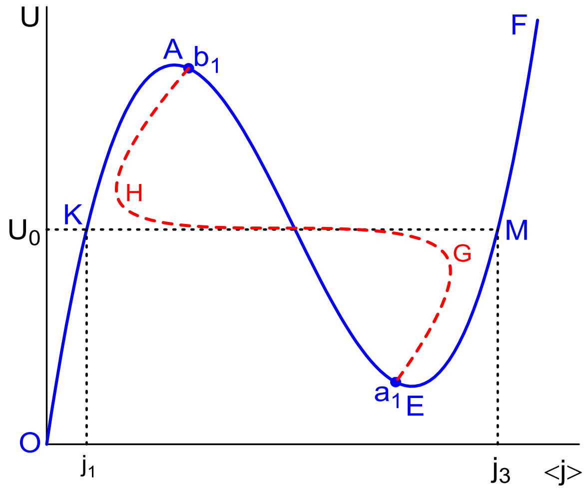

Figure 6. Schematic of the CVC of a mode with a normal spot.

The coexistence of phases typically occurs only at a certain value of the control parameter, in this case, of the voltage drop U_0. This value is independent of W and governed by a condition of solvability of a planar boundary-value problem governing the domain wall. Such solvability conditions are known as Maxwell's constructions; e.g. [21]. A derivation of an explicit form of Maxwell's construction for a particular problem is not a simple task. In the problem considered, an explicit form of Maxwell's construction or, in other terms, an explicit condition governing the normal voltage has been derived for arc cathodes [9, 20, 22] but not for glow discharges.

The evolution of the current density distribution from smooth (harmonic) in the vicinity of the states a_1 and b_1 to spot-like at states G and H is described in [20], as well as the interaction of a normal spot with lateral boundaries and/or other spots (in particular, this interaction is responsible for the correlation between the shape of the normal cathode spot in glow discharges and the shape of the discharge tube cross section). The difference $U - U_0$ in regimes with a normal spot is exponentially small, hence the corresponding section of the CVC (section HG in figure 6) is close to the horizontal line even if the ratio L/W is not very small.

In the problem considered, the domain wall separates the normal spot from the surrounding current-free region and its characteristic length scale L is represented by the thickness of the near-cathode space-charge sheath in the case of the glow discharge and by the cathode height in the case of the arc cathode. The thickness of the sheath is in many cases much smaller than the radius of the discharge tube, therefore regimes with normal spots on glow cathodes are usual. In contrast, thermionic cathodes of high-pressure arcs are in many cases thin rather than wide and regimes with normal spots usually do not occur.

2.4. Different physical mechanisms or multiple solutions?

Many authors have tried to explain the existence of different modes of current transfer to electrodes of gas discharges as a manifestation of different physical mechanisms. For example, thermionic versus thermo-field or field mechanisms of emission of electrons by the cathode surface were invoked to explain spots and the diffuse mode on cathodes of high-pressure arcs; e.g. [23–25]. The authors [26–28] considered that the basic mechanisms of a glow discharge are insufficient to explain spot patterns observed in glow microdischarges and additional mechanisms are needed, such as an increasing dependence of the effective secondary emission coefficient on the reduced electric field and heating of the gas. Furthermore, it was suggested that the regular arrangement of the developed spots (filaments) may be explained by assuming a balance of electrostatic forces: the positive charge of the cathode fall in one filament is subject to repulsive forces from other filaments, which are balanced by electrostatic forces due to surface charges deposited on the surface of the surrounding dielectric spacer.

We will not discuss here the relevance of particular mechanisms proposed in the above-cited works. Rather, we stress that different modes of current transfer are not necessarily a manifestation of different physical mechanisms: if a spot or pattern is unrelated to non-uniformities of the electrode surface, then it is a manifestation of self-organization. Hence, an adequate theoretical model should admit multiple solutions which exist at the same discharge current and describe a spotless mode and mode(s) with spots or patterns. What is really needed in the model is a positive feedback strong enough so that the CDVC of the spotless mode be N-shaped under conditions of interest, and usual mechanisms of near-cathode space-charge sheath illustrated by figure 4 are sufficient to this end.

3. Computation of multiple solutions

3.1. Straightforward approach

Suppose that one needs to compute a solution describing a mode with, say, a spot at the center of the cathode (or with a ring spot, or with two spots at the edge and so on). If an appropriate multidimensional code simulating a dc glow discharge or cathodic part of a high-pressure arc discharge is available, one can try to proceed in a straightforward way: to specify an external circuit, to choose an initial state resembling the desired solution, and to start the code.

Papers reporting multiple solutions obtained by means of such a straightforward approach exist, but are few. Axially symmetric current transfer to a cylindrical arc cathode was modeled in [29]. A unique solution was found for a cathode geometry corresponding to experimental conditions; however, two solutions were found in a certain current range for a wide cathode, one of these solutions describing the diffuse mode and the other the spot mode. The simplest axially symmetrical patterns on glow anodes (a single spot, a central spot surrounded by a ring spot) have been computed in [30]. Steady-state solutions with one or two filaments, depending on the initial conditions, have been obtained in time-dependent modeling of a planar glow discharge [31]. Steady-state solutions with filaments have been obtained in planar time-dependent modeling of the cathode layer of a non-self-sustained glow discharge in the subnormal regime [32] and of a thin glow discharge sandwiched with a semiconductor layer under cryogenic conditions [33]. The formation of self-organized anode patterns in a dc atmospheric-pressure free-burning arc discharge was successfully simulated in [34, 35]. In most cases, the reported solutions refer to just one current value or to a narrow current range. The full region of existence of each solution was not explored in any of the works.

These examples show that the straightforward approach to computation of multiple solutions in the theory of dc glow discharges and the cathodic part of arc discharges can succeed. As the pattern of multiple solutions and of their stability becomes more clear, the rate of success of this approach will certainly increase. On the other hand, it is desirable to develop also an approach that will allow one, at least in principle, to compute all steady-state solutions existing in a particular problem in a systematic way. Such an approach is considered in the next section.

3.2. Systematic approach

3.2.1. The idea There are two difficulties that one should overcome in order to be able to compute multiple solutions in a systematic way. First, one needs to know in advance that the particular solution being sought does exist under conditions specified. Indeed, if in the framework of the straightforward approach the code has returned a steady-state solution which is not the one being sought or has failed to return any solution, one will not know whether this is a numerical problem or the solution being sought does not exist for the external circuit specified, or maybe does not exist at all for the discharge conditions being considered.

Second, steady-state solutions in gas discharge physics are virtually universally computed by means of time-dependent codes: an initial state of the discharge is specified and its evolution over time is followed until a steady state has been attained. Such codes can compute only those steady states which are stable for the specified external circuit against perturbations having the symmetry to which the code is adjusted. It should be stressed that this kind of stability is not equivalent to physical stability: for example, a time-dependent code may fail to find a steady state which can be observed in a current-controlled discharge if the ballast resistance specified in the modeling is not sufficiently high; a steady state found by an axially symmetric time-dependent code may be unstable against 3D perturbations (which are usually the most dangerous ones).

Furthermore, even if the above-described kind of stability were equivalent to physical stability, difficulties would arise when this approach is applied in practice. A steady-state multidimensional solution is in many cases unstable in the vicinity of the bifurcation point where it branches off, but is stable beyond this vicinity; e.g. [6, figure 10]. Furthermore, stable and unstable sections of the same mode may alternate; see, e.g., [36, figure 1] for glow discharges and [37, figure 7] for arc cathodes. This impedes computing stable sections in a systematic way without computing also unstable sections. And even if a systematic computation of only stable steady states were possible, it would produce a fragmentary picture: a patch of solution here and another there, and one would be unable to understand the overall pattern. In fact, one would have difficulties even in identifying patches belonging to the same mode.

In more general terms, the pattern of multiple solutions may be complex enough and the pattern of their stability is still more complex, especially in the case of glow discharges; hence, one would prefer to decouple computation of steady-state solutions and analysis of their stability.

The above difficulties can be overcome in the following way. The bifurcation analysis can be employed in order to find which steady-state solutions exist and what is the region of existence of each one. The computation of steady-state solutions and the analysis of their stability can be decoupled by resorting to a steady-state solver (i.e. a one which computes a solution of discretized steady-state equations by means of an iteration process which is not equivalent to relaxation in time): one will first use a steady-state solver in order to compute all steady states which constitute a given solution (mode) regardless of whether each particular state is stable or unstable for some or the other external circuit, and study the stability of each state at a later stage.

3.2.2. Procedure. The modeling starts in the framework of a model of the particular discharge which, while being multidimensional, admits a 1D solution, as shown in figure 2. The first step consists in computing the 1D solution. If the CDVC described by this solution is N-shaped under conditions of interest, then one can hope that the mechanisms accounted for in the model are indeed sufficient for the model to admit multidimensional solutions describing modes with spots. If the CDVC is of another shape, e.g. monotonically growing, this is an indication that no multidimensional solutions probably exist and the only existing solution is 1D and describes the spotless mode.

The second step consists in finding points of primary bifurcations, i.e. 1D states where the increment of multi-dimensional perturbations of any mode vanishes. If the analysis has shown that the increment of multidimensional perturbations of a given mode is non-zero for all 1D states on the falling section of the CDVC, then the corresponding steady-state multidimensional solution may not exist. If there are no bifurcations of any multidimensional perturbations (which happens if the cathode is very thin), this is again an indication that the only solution existing under the considered conditions is 1D and describes the spotless mode.

Steady-state multidimensional solutions branching off from the 1D solution are computed in the third step. The computation of each solution starts at a value of the control parameter for which the solution for sure exists; it can be a value corresponding to the vicinity of one of the bifurcation points or any value between the bifurcation points. (The control parameter may be U, i.e. the discharge voltage in the case of the glow discharge and the near-cathode voltage drop in the case of the arc cathode; or discharge current I; or one of the parameters of the external circuit.) Then the solution is extended, by gradually varying the control parameter and using as an initial approximation the solution found for the previous value of the control parameter, with the aim to compute all steady states which constitute a given solution (mode) in the whole region of existence regardless of their stability.

After the second and third steps have been completed, solutions of the second generation are known. If there are reasons to believe that solutions of the third and next generations exist and are of interest, as is the case of glow microdischarges [38], then one can repeat these steps as necessary, finding points of secondary bifurcations and third-generation solutions, then points of tertiary bifurcations and fourth-generation solutions, and so on.

Normally, one will be interested not in the simplified discharge configuration shown in figure 2 but in a more realistic one. Then the configuration is gradually changed from that shown in figure 2 to the one of interest and the evolution of each of the multiple solutions is followed; the fourth step.

The fifth step consists in the analysis of the stability of the computed multidimensional solutions, with the aim to find which states are stable and may realize in the experiment. Note that this step is very important since each solution of a problem with multiple solutions is usually unstable at least in a part of its existence region.

The above approach allows one, at least in principle, to compute multiple steady-state solutions and their stability in a systematic way. Of course, the approach is laborious. On the other hand, there is no need every time to perform the whole procedure in full after the pattern of multiple solutions has been found. For example, if one needs to compute a 3D spot pattern in an axially symmetric geometry different from the one shown in figure 2, then a first-guess shortcut is to compute the simplest 2D mode (the fundamental mode; see section 4.2) and, if its CVC is N-shaped, to run a stationary 3D code somewhere on the falling section of the CVC with an initial approximation resembling the desired pattern. The technique of computation of different 2D solutions for the case of the arc cathode without previously finding bifurcation points is given in section 4 of the tutorial of the tool [39].

3.2.3. Numerical aspects. From the point of view of numerics, the first above-described step poses no difficulties. In particular, steady-state 1D solutions describing current transfer to cathodes of glow discharges have been computed by many researchers in the framework of many different models and physics behind these solutions has been generally well understood. One should only ensure that the code being used can compute steady states corresponding to the falling section of the CDVC.

The second step is performed with the use of the formalism of the linear stability theory. Multidimensional eigenvalue problems governing primary bifurcations admit separation of variables and may be reduced to 1D eigenvalue problems. In the case of an arc cathode, the latter problem admits an analytical solution [9]. In the case of a glow discharge, the latter problem can be readily solved numerically; a home-made solver was used in [18] and an eigenvalue solver provided as a part of commercial software COMSOL Multiphysics was used in [8, 40]. 3D eigenvalue problems governing bifurcations of 3D solutions from axially symmetric ones admit separation of variables and may be reduced to 2D eigenvalue problems, which have been solved numerically by means of a home-made solver [39, 41] (for arc cathodes) and an eigenvalue solver of COMSOL Multiphysics [36, 40]. 3D eigenvalue problems governing branching of 3D solutions have been solved for arc cathodes by means of the eigenvalue solver of COMSOL Multiphysics [8, 40].

The third step requires a multidimensional solver which computes a solution of discretized steady-state equations by means of an iteration process which is not equivalent to relaxation in time. Good results have been obtained with steady-state solvers based on the Newton linearization with a direct solution of linear equations in finite elements or differences, such as steady-state solvers of COMSOL Multiphysics (e.g. [42–46]) or a home-made steady-state solver used in [39, 47]. Note that such solvers indeed allow one to decouple issues of numerical and physical stability; for example, one can compute without any difficulty states on falling branches of CVCs treating U as a control parameter, i.e. without a ballast resistance required by time-dependent solvers. Note also that steady-state solvers usually allow using more refined meshes, which may be important, especially for the glow discharges. Also used was the commercial finite-element platform ANSYS [48–51].

Specifying an initial approximation for beginning of calculation of a desired multidimensional solution is a delicate point. A safe way is to begin in the vicinity of a bifurcation point where the multidimensional solution in question branches off from the 1D solution and use as an initial approximation a superposition of the 1D solution corresponding to the bifurcation point and a small harmonic perturbation. In some cases the solver spontaneously switches to the desired solution provided that the necessary arrangements have been made, an example being computation of solutions appearing through period-doubling bifurcations in the modeling of a glow discharge [38].

Hints on calculation of multiple solutions on arc cathodes are given in [6, section 3.3.2]. These hints are also applicable to the calculation of glow discharges. It happens in the modeling that iterations, having converged for one value of the control parameter, fail to converge for the next value, however small the increment of the control parameter is. Since a solution can turn back or join another solution but cannot just disappear, such a break-off represents a failure of the method. The most frequent reason is that an extreme point of the CVC or a turning point has been encountered: a code cannot pass through these points if operated with, respectively, U or I as a control parameter. An obvious fix is to switch the control parameter. It is essential, therefore, that the code allows to make this switching in an easy and seamless way. The second most frequent reason is the numerical mesh being not fine enough. Another point discussed in [6, section 3.3.2] concerns computation of 3D solutions in axially symmetric calculation domains.

The fourth step may be performed by means of the same solver as the third step.

An investigation of stability of steady states in the fifth step may be performed by means of the formalism of the linear stability theory. The arising multidimensional eigenvalue problem may be solved numerically by means of an eigenvalue solver of COMSOL Multiphysics [36, 52]; for the arc cathode also analytical results are available [19]. Relevant aspects of the use of the eigenvalue solver of COMSOL Multiphysics are discussed in [36, 40] and references therein. An alternative to the usage of the linear stability theory is to employ a time-dependent code: the code is run with the initial condition being the steady state in question on which a small perturbation is superimposed, and the time evolution of the perturbation is observed. This approach is significantly more demanding in terms of CPU time. On the other hand, it allows one to follow also the nonlinear stage of development of the instability.

A 2D simulation technique of arc plasma–cathode interaction has reached a point where it can be automated. A free online tool for simulation of multiple modes and bifurcations of axially symmetric steady-state current transfer to rod thermionic cathodes in high-pressure plasmas is available on the Internet [39]. The tool is accompanied by a tutorial that would help applied physicists and engineers working in the field to make themselves comfortable with multiple solutions describing different modes of current transfer. The experience gained with the tool will facilitate computing different modes of current transfer by means of other tools, such as COMSOL Multiphysics or ANSYS.

4. Available results

4.1. Idealized geometry

Figure 7 depicts multiple steady-state solutions computed for a glow discharge and plasma–cathode interaction in a high-pressure arc discharge in the idealized geometry which admits 1D solutions and is shown in figure 2. Here R is the discharge radius and h is the interelectrode distance in the case of the glow discharge and R is the radius of the cathode and h is its height in the case of the arc cathode. Each solution is illustrated by a typical distribution of current density over the cathode surface (red means the highest value and blue the lowest), which gives an idea of the spot pattern associated with the mode of current transfer described by this solution. Note that the spot pattern varies with current, therefore images shown in figure 7 and similar figures below are representative of some but not all steady states described by the corresponding solution.

The line NP in figures 7(a) and (b) represents the CDVC described by the 1D solution. (Note that the line NP in figure 7(a) represents the same data as the solid line in figure 1.) The current density distribution over the cathode surface described by this solution is uniform; the spotless mode. The CDVC is falling for $\langle j \rangle \leqslant 330\,\mathrm{A\,m^{-2}}$ and

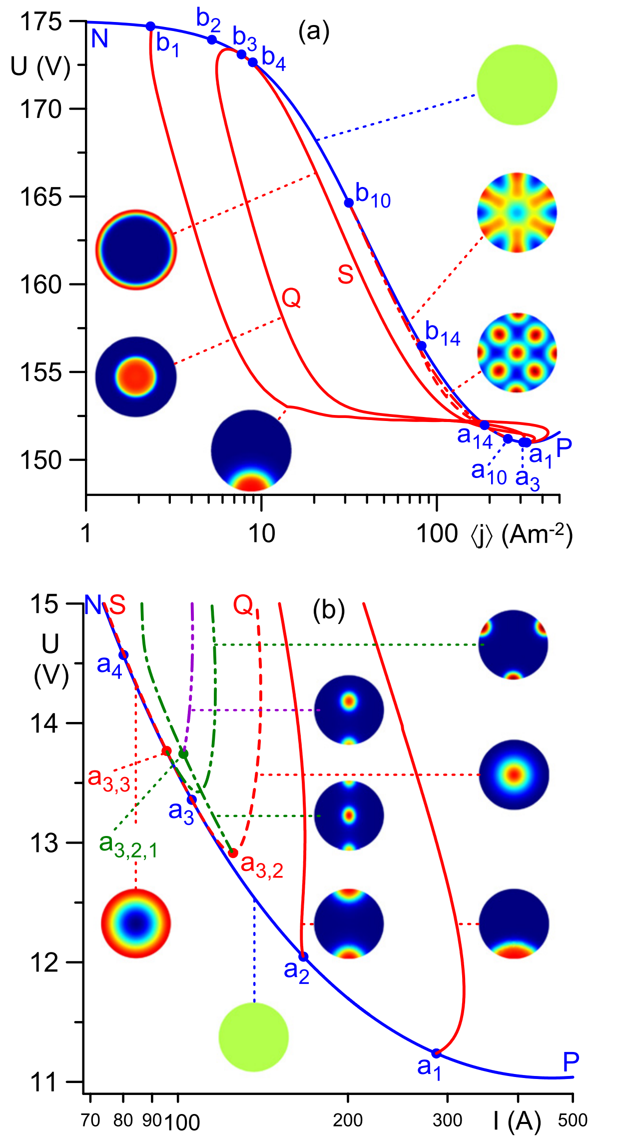

Figure 7. CVCs and schematics of current density distribution over the cathode surface described by different steady-state solutions. Geometry shown in figure 2. (*a*) Glow discharge, Xe plasma, $p = 30$ Torr, $R = h = 0.5$ mm. Data from [7, 53]. (*b*) Cathode of an arc discharge; Ar plasma, $p = 1$ bar, W cathode, $R = 2$ mm, $h = 10$ mm. Adapted from [8].

$I \leqslant 460$ A, respectively, and growing for higher currents, i.e. the CDVC depicted by the line NP is U-shaped. However, this line represents not a full CDVC but rather just one branch of it: the full CDVC also includes a branch corresponding to the situation where no discharge is present. The latter branch is described by the trivial solution of the considered problem: the ion and electron densities are zero in the discharge tube and the applied electric field is unperturbed in the case of the glow discharge; the temperature in the cathode body is constant and equal to that of the cooling fluid in the case of the arc cathode. In the case of the glow discharge, this branch coincides with the section of axis of voltages up to the breakdown voltage. In the case of the arc cathode, this branch coincides with the whole axis of voltages since the model being employed ([6] and references therein) does not account for the possibility of breakdown and the voltage $U(j)$ given by the model infinitely increases for small j.

In other words, the full CDVC includes not only the (U-shaped) branch representing the characteristic of the discharge itself, but also the branch coinciding with the axis of voltages, or a part of it, and representing the situation where the discharge has not been ignited. Thus, the full CDVC is similar to the schematic $OAEF$ in figure 3, the difference being that the state A under conditions of figure 7 belongs to the axis of voltages and in the case of the arc cathode is positioned at voltages infinitely high. The full CDVC is thus N-shaped rather than U-shaped.

In terms of section 2.2, the 1D solutions shown in figures 7(*a*) and (*b*) by the line NP represent the first generation. Multidimensional solutions of the second generation branch off from the 1D solution at the bifurcation points a_1, a_2, a_3, a_4, ... The positions of these bifurcation points and the character of bifurcating solutions are related to zeros of the derivatives of the Bessel function of the first kind of different orders [8]. In particular, the solutions branching at a_1, a_2, a_4, a_5, a_7, a_{10}, a_{13}, a_{15} are 3D with azimuthal periods of 2π, π, $2\pi/3$, ..., $\pi/4$, respectively, and typically describe patterns with one to eight symmetrically positioned spots on the periphery of the cathode. Other solutions with a period of 2π branch off at a_6 and a_{12}. Other solutions with periods of π, $2\pi/3$ and $\pi/2$ branch off at a_8, a_{11} and a_{14}, respectively. Solutions branching off at a_3 and a_9 are 2D. In the case of the glow discharge, the second-generation solutions rejoin the 1D solution at the bifurcation points $b_1, b_2, b_3, b_4, \ldots$, respectively. Only some of the second-generation solutions are shown in figures 7(*a*) and (*b*). Solutions shown in figure 7(*a*) are a_1b_1, a_3b_3, $a_{10}b_{10}$ and $a_{14}b_{14}$, although the CVCs of the solutions $a_{10}b_{10}$ and $a_{14}b_{14}$ are close to the line NP and are barely visible. The solutions branching off at the points a_1, a_2, and a_3 are shown in figure 7(*b*); however, the most part of the dashed line Sa_3Q representing the latter solution virtually coincides with the line NP and only the section $a_{3,2}Q$ is visible.

The 2D solution that branches off at a_3 and (in the case of the glow discharge) rejoins the 1D solution at b_3 comprises two branches separated by the bifurcation point(s): branch a_3Qb_3 (or, in the case of the arc cathode, a_3Q), associated with a spot at the center of the front surface of the cathode, and branch a_3Sb_3 (or, in the case of the arc cathode, a_3S), associated with a ring spot on the periphery.

The solutions described by lines $a_{10}b_{10}$ and $a_{14}b_{14}$ in figure 7(*a*) are associated with many-spot patterns, which vary with current, as shown in [53]. It is seen from figure 7(*a*) that the plane ($\langle j\rangle$, U) is not suitable for representation of these solutions: their CVCs virtually coincide with that of the 1D solution. Adequate and convenient are coordinates ($\langle j\rangle$, j_c), where j_c is the current density at the center of the cathode. (One could also use the current density at a fixed point at the edge of the front surface of the cathode instead of j_c, as discussed in [8].) This representation is used in figure 8. One can see that different solutions are indeed clearly visible in this figure.

In the case of the glow discharge, CVCs described by the first 3D solution a_1b_1 and by the central-spot branch a_3Qb_3 of the first 2D solution in figure 7(*a*) manifest a plateau,

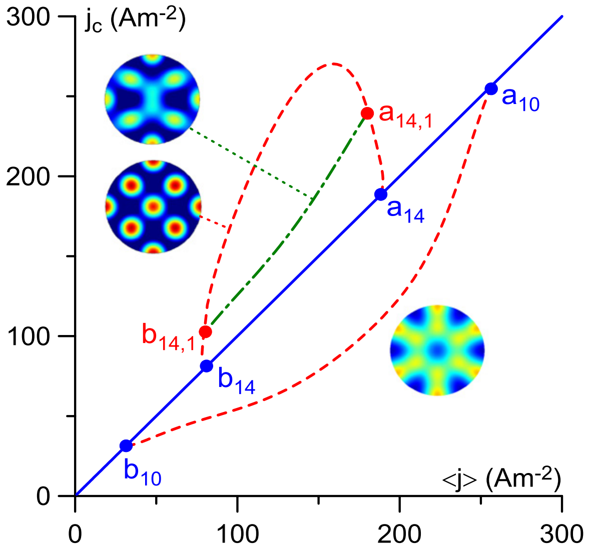

Figure 8. 1D solution and 8th and 12th 3D solutions. Geometry shown in figure 2(a); glow discharge in Xe plasma, $p = 30$ Torr, $R = h = 0.5$ mm. Data from [54].

similarly to the schematic shown in figure 6. These are regimes with coexistence of phases or, in other terms, with normal spots, illustrated by figure 5. The occurrence of these regimes on glow cathodes but not on arc cathodes, revealed by the numerical results, is a consequence of different aspect ratios as discussed at the end of section 2.3. The multi-spot modes $a_{10}b_{10}$ and $a_{14}b_{14}$ for the glow discharge do not reveal regimes with normal spots, again in agreement with the discussion in section 2.3.

Also shown in figures 8 and 7(b) are examples of 3D solutions of the third generation. The dotted–dashed line $a_{14,1}g_{14,1}$ in figure 8 represents a solution with the period of π which branches off from the mode $a_{14}b_{14}$ through period-doubling bifurcations occurring at states $a_{14,1}$ and $b_{14,1}$. The dotted–dashed and two dotted–dashed lines in figure 7(b) represent 3D solutions with azimuthal periods of, respectively, π and $2\pi/3$ which branch off from the 2D solution Sa_3Q through bifurcations occurring at states $a_{3,2}$ and $a_{3,3}$, respectively. Note that the absence of a central spot in patterns associated with the mode branching off at $a_{3,3}$ is a consequence of the bifurcation point $a_{3,3}$ being positioned on the branch without a central spot (a_3S) of the 2D solution. It is interesting to note also that no bifurcation of a 3D solution with a period of 2π was detected on the mode Sa_3Q (it is for this reason that the designation $a_{3,1}$ is not used in figure 7(b)) and the bifurcation point $a_{3,2}$ coincides with the point of minimum of the CVC Sa_3Q [41].

The three dotted–dashed line in figure 7(b) depicts a fourth-generation 3D solution with a period of 2π, which branches off from the above-described third-generation solution with a period of π through a period-doubling bifurcation occurring at state $a_{3,2,1}$.

The period-doubling bifurcation that occurs at $a_{14,1}$ and $b_{14,1}$ in figure 8 is accompanied by (or, as one can say, occurs through) splitting of the central spot. The period-doubling bifurcation that occurs at $a_{3,2,1}$ in figure 7(b) occurs though a change in the central spot as well: the central spot in the three-spot configuration existing at $a_{3,2,1}$ starts moving upwards and this movement is accompanied by the extinction of the upper peripheral spot and by an enhancement of the lower peripheral spot, so the two-spot pattern shown in the image in figure 7(b) appears.

Of course, patterns of stationary spots shown in figures 7 and 8 do not exhaust all possibilities. Some further examples for glow discharges can be found in [38, 54]. In particular, another scenario of period doubling may occur for patterns with ring(s) consisting of an even number of spots: every other spot in each ring becomes different from its neighbors. For example, it can move in the radial direction; see states d_1 and A in [38, figure 10a], or states d_2 and E in [38, figure 10b] (note that states d_1 and d_2 of [38] are denoted $a_{10,1}$ and $a_{10,2}$ in designations of this work). Other possibilities are a shift in the azimuthal direction and variations in brightness.

Most of the results for glow discharges available to date, including those shown in figures 7(a) and 8, have been computed in the framework of the simplest self-consistent model, which accounts for a single ion species produced via a single effective ionization process and employs the local-field approximation. It is important to stress in this connection that an account of detailed plasma chemistry and non-locality of electron kinetics results in an increase in the number of multiple solutions but does not change their pattern [38].

One can conclude that the pattern of computed multiple solutions conforms to the one established theoretically in sections 2.2 and 2.3. In particular, figures 7(a) and (b) are qualitatively similar to the schematic shown in figure 3: there are 2D and 3D solutions bifurcating from the 1D spotless mode (second-generation solutions) and these bifurcations occur on the falling section of the CDVC of the spotless mode; 3D solutions of next generations appear through sequential bifurcations. A difference is that, while the second-generation multidimensional solutions at low currents rejoin the 1D solution in the case of glow discharges, they do not in the case of arc cathodes. It is intuitively clear that this difference originates in the voltage $U(j)$ infinitely increasing for small j in the model of arc–cathode interaction being employed and will disappear if the account of glow-to-arc transition has been introduced.

The reasoning of sections 2.2 and 2.3 is not rigorous, hence one should not be surprised if its conclusions are not completely correct in some cases. Indeed, it has been found [7, section 3.2] that there is a narrow range of conditions of the glow discharge where a second-generation 2D solution still exists while the corresponding bifurcations on the 1D solution have already disappeared; primary bifurcations in the case of the glow discharge computed with diffusion of the charged particles being neglected compared to drift may be positioned on the beginning of the growing section of the CDVC (section EF in figure 3) rather than on the falling section [18], [7, section 3.3]. However, these cases are marginal; the theory of sections 2.2 and 2.3 correctly describes the picture on the whole and is accurate enough to be useful.

4.2. Effect of lateral wall

The pattern of multiple solutions seen in figures 7 and 8 is the simplest from the theoretical point of view since it has been computed in the idealized configuration admitting a 1D solution. In order to obtain a model relevant to the experiment, one should take into account effects introduced by the lateral wall: absorption of the ions and electrons by the lateral wall of the discharge tube with their subsequent recombination in the case of a glow discharge and collection of electric current and energy flux from the plasma by the lateral surface of an arc cathode. One can expect that this will affect the pattern of multiple solutions in two ways. The first change is obvious: an absorbing wall of a discharge tube reduces the intensity of a glow discharge in the vicinity of the wall due to losses of the charged particles caused by diffusion to the wall; in contrast, energy- and current-collecting lateral surface of an arc cathode provides an additional heating of the edge of the front surface and thus locally enhances the discharge. As a consequence, 3D spots originally positioned in contact with the cathode edge will be shifted inside the cathode in the case of the glow discharge and will expand to the lateral surface of the cathode in the case of the arc cathode. This is indeed the case, as shown in [38] for the glow discharge and [43] for arc cathodes.

The second change originating in effects introduced by the lateral wall concerns 2D solutions and is as follows. The model with account of these effects does not admit 1D solutions. However, it does admit a 2D solution which is in some aspects analogous to the 1D solution in the model with the idealized geometry; in particular, both exist at all discharge currents and 3D solutions branch from both. Let us designate the mode described by this 2D solution the fundamental mode. 2D solutions do not branch from the fundamental-mode solution: the latter solution is 2D as well, i.e. has the same symmetry, and no breaking of symmetry normally means no bifurcations. Hence, bifurcations occurring in the original model at states a_3 and b_3 in figure 7(*a*) and a_3 in figure 7(*b*) disappear in the modified model and the fundamental mode becomes separated from all the other 2D modes (i.e. from 2D modes existing in a limited current range; we will call such modes non-fundamental 2D modes). The phenomenon of destruction of a bifurcation and breaking of the bifurcating solutions into two isolated solutions is well known; a self-sufficient summary of relevant information from the bifurcation theory can be found in [8, appendix A3].

In particular, it is known that a destruction of a bifurcation is accompanied by exchange of branches of the solutions. This means that the sections Pa_3 and (in the case of the glow discharge) Nb_3 of the 1D solution separate from the section a_3b_3 (or, in the case of the arc cathode, from the section a_3N) and join one of the branches of 2D solution bifurcating at a_3 and (in the case of the glow discharge) b_3. In agreement with what was said above concerning the first change, the glow discharge on the sections Pa_3 and Nb_3 will be less intense on the periphery of the cathode than at the center, and one should expect that these sections will join the branch a_3Qb_3, which has similar properties. In contrast, the arc discharge on the section Pa_3 will be more intense on the periphery than at the center, hence one should expect that this section will join the branch a_3S.

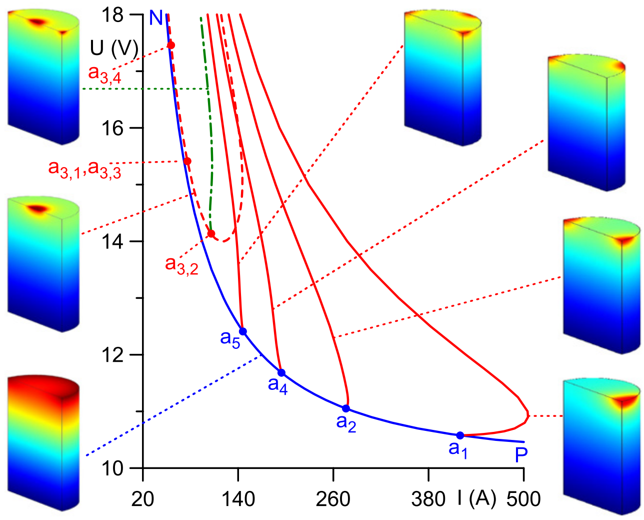

Figure 9. CVCs and schematics of current density distribution over the surface of a cylindrical arc cathode described by different steady-state solutions;Ar plasma, $p = 1$ bar, W cathode, $R = 2$ mm, $h = 10$ mm. Adapted from [6].

Thus, the second change may be described as follows: the fundamental-mode solution, i.e. the 2D solution which exists at all discharge currents in the model with active lateral surface, has as its analog in the original model not the 1D solution NP but rather a composed solution Nb_3Qa_3P (or, in the case of the arc cathode, Sa_3P), which comprises both 1D and 2D sections. A more detailed discussion can be found in [8].

As an example, let us consider figure 9, which has been computed for the same conditions as figure 7(*b*) but with account of collection of electric current and energy flux by the lateral surface of the cathode. The line NP represents the fundamental-mode solution. 3D solutions with one to four spots on the edge of the cathode branch off from the fundamental-mode solution at the states a_1, a_2, a_4 and a_5. The dashed line represents a non-fundamental 2D solution, from which another four 3D solutions branch off at the states $a_{3,1}$ to $a_{3,4}$. (The bifurcation point $a_{3,3}$ coincides with $a_{3,1}$ to the graphic accuracy; only shown is the 3D solution that branches off at the bifurcation point $a_{3,2}$.) The pattern of multiple solutions seen in figure 9 is similar to that of figure 7(*b*) except for the above-described changes: spots on the periphery of the front surface extend to the lateral surface; the fundamental-mode discharge is more intense at the edge than at the center; the non-fundamental 2D solution is detached from the fundamental-mode solution. Note that bifurcation points a_1, a_2 and $a_{3,2}$ in figure 7(*b*) correspond to those in figure 9; bifurcation occurring at the state a_3 in figure 7(*b*) has no analog in figure 9 (was destroyed), which is why the designation a_3 is not used in figure 9; bifurcation points a_4 and $a_{3,3}$ in figure 7(*b*) do not correspond to those in figure 9 because of the above-described exchange of branches.

The exchange of branches is not manifested in figure 9, the reason being that the CVCs of the 1D solution and the composed solution, NP and Sa_3P in figure 7(*b*), are quite close. However, the exchange of branches does manifest itself in numerical simulations. Suppose that one wishes to compute the fundamental-mode solution by starting from the 1D solution and then gradually eliminating the insulation from the lateral surface of the cathode at a fixed voltage until a solution for a fully active lateral surface has been found. (This is one of the options offered by the online tool [39]; the example being described was taken from the tutorial available at the site.) This approach works if the initial state belongs to the section Pa_3 of the 1D mode. If one starts from a state on the section a_3N, then the iterations either converge to a state belonging to the low-voltage branch of the non-fundamental 2D solution, or diverge. Further details can be found in [8].

In the case of the glow discharge, the CVCs of the 1D solution and the composed solution, NP and Nb_3Qa_3P in figure 7(a), are not close, therefore the exchange of branches in the case of the glow discharge should be readily visible. This is indeed the case, as shown by figure 1. The above reasoning explains how the diffusion losses of the ions and the electrons to the wall, which is a weak effect, can originate such a large difference between the 1D solution in the model with a reflecting wall and the fundamental-mode 2D solution in the model with an absorbing wall: the diffusion losses can significantly affect only states where the balance is delicate, which are bifurcation points, and result in a destruction of bifurcations; the subnormal and normal sections are already present in the model with the reflecting wall, but they represent a part of the first 2D solution rather than of the 1D solution; they become a part of the fundamental-mode solution as a result of the exchange of branches accompanying the destruction of bifurcations.

4.3. Stability of multiple solutions

Of course, only some of the above-described steady-state solutions are observed in the experiment, which means that most of them are unstable. Therefore, investigation of their stability is of primary importance.

The eigenvalue problem governing the stability of steady-state solutions in the framework of the linear stability theory is Hermitian (self-adjoint) in the case of arc cathodes [19], which means that its spectrum is real. This is not in the case for the glow discharge. Therefore, stability analysis is significantly more difficult in the latter case than in the former.

The pattern of stability of steady-state current transfer to arc cathodes has been investigated analytically [19] and numerically [52]. It was shown, in particular, that the stability of the fundamental mode conforms to the reasoning on the stability of the 1D spotless mode given in section 2.2. Results for the case of a current-controlled arc on a rod cathode may be summarized as follows: modes with a spot at the center or with multiple spots are always unstable; the only modes that can be stable are the fundamental mode and the high-voltage branch of the first 3D spot mode; the transition between these two modes is non-stationary without oscillations and accompanied by hysteresis.

The stability of glow microdischarges was investigated numerically [36]. The fundamental mode is stable in the abnormal regime, in a certain current range in the normal regime, and at low currents in the Townsend regime. The loss of stability in the Townsend regime is likely to occur in an oscillatory way and the loss of stability in the abnormal regime is likely to occur in a monotonic way. Of all modes with patterns, stability has been investigated of only the first and second 2D modes. It is found that the first 2D non-fundamental steady-state mode is stable in a certain current range on the high-voltage branch.

A comparative analysis of the stability of current transfer to cathodes of vacuum and ambient-gas arc discharges, including the nonlinear stage of evolution of unstable states, was performed by means of a time-dependent code [55].

4.4. Experimental validation

By now, a theory of diffuse and spot modes on cathodes of high-pressure arc discharges based on the concept of multiple solutions has gone through a detailed experimental validation by means of different methods, such as spectroscopic measurements, electrostatic probe measurements, electrical and pyrometric measurements, and calorimetry. The effort was focused on low-current arcs typical of high-intensity discharge lamps. One can specifically mention works of Mentel and co-workers, in particular, [42]. Further references can be found in [6]; one can also mention works [45, 56, 57] as more recent examples. In particular, in [56] the possibility of real-time quenching of the instability causing the formation of spots on cathodes of high-pressure arc discharges was demonstrated by means of numerical simulations and experimentally. This possibility stems from the fact that the instability is of thermal nature and therefore slow.

Multiple solutions computed in the theory of glow discharges agree with the experiment as well, although the comparison has been merely qualitative up to now. In particular, the composed solution Nb_1a_1P in figure 7(a), which describes the 2D abnormal discharge at high currents, the 2D Townsend discharge at low currents, and the normal discharge with a 3D normal spot attached to the edge of the cathode at intermediate currents, conforms to experimental information cited in textbooks on gas discharges.

Spot patterns computed in [7, 38, 53, 54], some of which are shown in figures 7(a) and 8, are similar to self-organized steady-state spot patterns observed during the last decade in experiments with glow microdischarges [26–28, 58–65], primarily by Schoenbach and co-workers. Note that the luminous objects observed in these experiments may be called filaments. However, they are produced in the cathode boundary layer, as shown, e.g., by the lateral photographs in [27]. Therefore, the term 'cathode spots' seems to be no less justified. Most observations [26–28, 58–65] have been performed in the so-called cathode boundary layer discharge (CBLD) electrode configuration, which comprises a planar cathode and a ring-shaped anode. However, it is important to stress that patterns observed in discharges with parallel-plane electrodes [60] are similar to those observed in the CBLD configuration.

A quasi-stationary, continuous and reversible transition between the 2D mode with a central spot and the 3D mode with four spots has been observed in the experiment [65, images (10)–(12) in figure 2]. These observations represent a direct proof of the existence of bifurcations of steady-state solutions, which is the cornerstone of the theory. The CVC of this transition [65, figure 3a] clearly represents a diagram of subcritical bifurcation, and so does also the CVC shown in [27, figure 3(a)]. Simulations of this transition can be found in [54].

The only type of bifurcations in which a 3D solution branches off from another 3D solution is period doubling as discussed at the end of section 2.2. It follows that a transition between, say, 3D patterns with 4 and 5 spots must be discontinuous and may be accompanied by hysteresis, as well as all the other similar transitions. Indeed, in the experiment such transitions occur in this way; e.g. [27, 65].

Until recently, the patterns were observed in experiments with dc glow microdischarges in 99.999%-pure xenon but not in other plasma-producing gases, such as argon [58] or krypton [27]. According to the theory, however, self-organization in dc glow discharges is a general phenomenon and not particular of xenon, and the modeling [38] indicated that multiple solutions exist in gases other than xenon provided that the pressure is high enough. Indeed, in recent experiments [64] patterns have been observed in krypton starting from pressures about twice as high as those necessary for self-organization in xenon, in agreement with the modeling predictions. Self-organization was also observed in xenon with 0.5% air impurity [65]. Note that the latter has been achieved by means of adjustment of the discharge current on the microampere scale, which also enabled the emergence of patterns that have not been observed previously; see below.

The conclusion of the modeling [36] that the loss of stability of Townsend and abnormal discharges is likely to occur, respectively, with and without oscillations, conforms to the experimental fact that oscillations can develop in the course of transition from the Townsend discharge to the normal glow discharge [10–17], while the transition from the abnormal discharge to the normal discharge or a steady-state mode with multiple spots normally occurs in a monotonic way. On the other hand, the modeling [36] showed that in a very narrow current range the loss of stability of an abnormal discharge occurs through perturbations oscillating in time, and this was observed in the experiment as well [27, figure 2(c)].

The conclusion [36] that the mode with a circular normal spot at the center of a (circular) cathode and the mode with a ring spot are stable in certain current ranges and therefore can be observed in the experiment, conforms to the experiment; see [27, figure 2 (b)] and [65, images 1–10 in figure 2] and, respectively, [65, figure 4]. Also observed in the experiment was a pattern comprising a central spot and a ring spot [65, figure 6(b)]. These results are particularly interesting since axially symmetric patterns, being prone to destruction by 3D perturbations, are by far more rare than 3D patterns; see, e.g., discussion in [66]. (As far as gas discharges are concerned, axially symmetric patterns have been observed on anodes of glow discharges [67, 68], in dielectric barrier discharges [66], and on liquid anodes of dc glow discharges [69].)

A transition from small spots to ring segments which subsequently merge into a ring observed in the experiment [65, figure 5] appears also in the modeling [54].

Thus, the concept of multiple solutions, when applied to the theory of glow discharges, allows one to understand and qualitatively describe in the framework of standard theoretical models, without invoking special mechanisms favoring self-organization, steady-state patterns of multiple spots and 3D and 2D normal cathode spots, as well as their transitions to the abnormal and Townsend discharges.

5. Related topics

5.1. Self-organization or geometrical concentrations of current?

There are two reasons for concentration of current in certain parts of the electrode surface, i.e. for the appearance of current spots: self-organization and non-uniformities of geometrical and/or physical properties of the surface, such as the presence of protrusions or areas with a reduced work function. The geometry shown in figure 2 represents a limiting case where the current-collecting surface of the cathode is perfectly uniform and the second reason is absent. As we have seen, there are multiple solutions in this case describing the spotless mode of current transfer and different spot modes. The spots are of a purely self-organization nature in this case. The opposite limiting case is the one where the cathode surface is strongly non-uniform and only one solution exists, meaning no self-organization.

A nice example of coexistence of patterns caused by self-organization and by geometrical non-uniformities is shown in figure 10, which is a photograph of a negative corona electrode of a spiral shape in an electrostatic precipitator [70]. A part of the cathode is dark with bright spots. These spots, or tufts, represent a self-organization phenomenon and are briefly discussed in section 5.5 below.

The other part of the cathode is bright with dark longitudinal stripes. These stripes seem to be caused not by self-organization but rather by the specific discharge geometry: the corona burns only on the side facing the grounded collection plate while the surface facing the inside of the spiral remains dark. Current-free stripes on cylindrical and spiral cathodes have also been obtained in 2D and 3D steady-state numerical simulations [71]. On the other hand, the bifurcation analysis [72] performed in the framework of the same model of a corona discharge [71, 73] that was used in [71] showed that there are no bifurcation points on the steady-state 1D cylindrically symmetric solution. This indicates that regimes with self-organized patterns do not exist in the framework of the model [71, 73]. Hence, the stripes computed in [71] (and observed in [70]) indeed represent a consequence of the specific discharge geometry rather than a self-organization phenomenon.

A transition between self-organized spots and geometrical current concentrations was studied numerically for high-pressure arc cathodes of different geometries [37]. The modeling started from a geometry in which there is a (self-organized) pattern of current transfer comprising two distinct

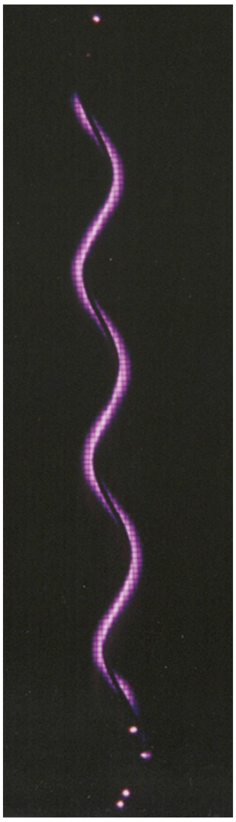

Figure 10. Negative corona electrode of an electrostatic precipitator. Courtesy of U Kogelschatz.

modes, the diffuse mode and the spot mode. As the geometry changes in a way that the degree of non-uniformity of the diffuse mode increases, a transition occurs to a pattern with a single mode, which embraces states with a diffuse temperature distribution at high currents and states with a hot spot at low currents. This transition is realized through a bifurcation of the diffuse mode and the spot mode, which occurs at a certain combination of control parameters, is not symmetry-breaking, and is accompanied by exchange of branches of the modes. In mathematical terms, this is a perturbed transcritical bifurcation of second-order contact [8].

5.2. Solitary cathode spots

The diameter of well-developed spots on glow cathodes in the experiment [27] was around 80 μm, which is comparable to the thickness of the near-cathode layer (between 50 and 70 μm, depending on the pressure and current) and much smaller than the cathode diameter (750 μm). The radius of spots on contacts of high-power vacuum circuit breakers does not exceed a few tens of micrometers, while the radius of a contact and the average distance between neighboring spots are of the order of 1 cm and 1 mm, respectively. These examples show that there are situations of practical interest where the spots are small. One can expect that an adequate theoretical description of a small spot may be obtained by means of neglecting the presence of other spots and the cathode boundaries. In other words, one can consider a solitary cylindrically symmetric cathode spot in a dc glow discharge between infinitely wide electrodes or on an infinite planar (half-space) arc cathode.

Modeling of solitary spots on a cathode of a high-pressure arc discharge was reported in [47]. Also studied in [47] was a transition from the spot mode on a finite cathode in the limiting case of large cathode dimensions to the solitary spot mode. Modeling of solitary spots on a cathode of a vacuum arc and of their stability was reported in [55, 74].

Approximate analytical models of solitary spots have been developed and used for several decades for modeling of cathode spots of vacuum arcs; e.g. [75–78]. Such models are based on dividing the cathode surface into a current-collecting circular region (a spot) of a constant temperature and the surrounding current-free region. The models are governed by two parameters, the spot temperature and radius, and the equation of integral heat balance of the spot is insufficient to determine both. Some models relied on empirical parameters, while in others the missing relation was obtained by invoking arbitrary theoretical assumptions, such as some or other implementation of Steenbeck's principle of minimum power or considerations concerning processes on the plasma side. An asymptotic solution exploiting the Arrhenius character of processes involved was found in [79] by means of the method of matched asymptotic expansions. The spot core, the current-free periphery and the transition region appear in the asymptotic analysis in a natural way as three different asymptotic regions. The spot radius is found to be governed by a condition of solvability of the problem describing the temperature distribution in the transition region. The spot model obtained in this way was tested on a model problem with a step-function dependence of the energy flux from the plasma to the cathode surface on the surface temperature [80] and gave useful results when applied to cathode spots in vacuum arcs [78].

5.3. Is there a place for Steenbeck's principle of minimum power?

Some researchers assume that if different modes of discharge are possible at the same discharge current, the mode with a lower voltage is the preferred (stable) one. This assumption stems from the so-called Steenbeck's principle of minimum power, which was proposed long ago for an arc discharge [81] and has been extensively invoked in investigations of many gas discharge phenomena, including the effect of normal current density on glow cathodes [1] and cathode spots in vacuum arc discharges [82]. One should bear in mind, however, that Steenbeck's principle is a not a corollary of the principle of minimum entropy production [83], in contrast to what is frequently assumed. Similarly, Steenbeck's principle is not a corollary of mathematical models of gas discharges; hence, this principle contradicts the mathematical models. It is not surprising, therefore, that the above-described results of investigations of the stability of multiple solutions have not confirmed the assumption of the mode corresponding to a lower voltage being preferable. Furthermore, the occurrence of a hysteresis in the modeling of, and experiments on, transition between the spot and diffuse modes on arc cathodes is by itself an unambiguous indication of incorrectness of this assumption.

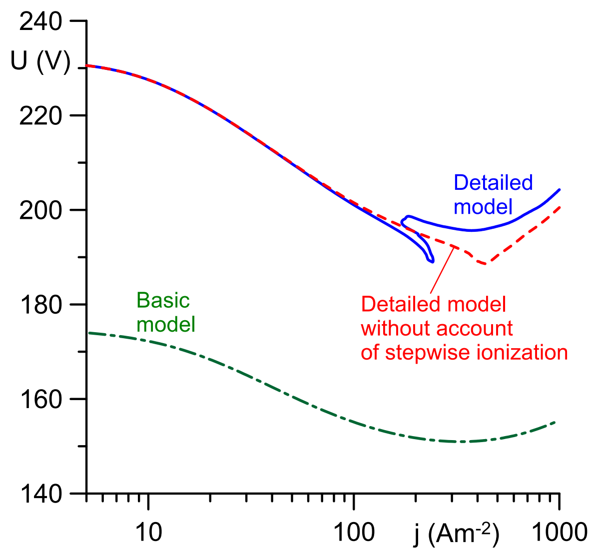

Figure 11. CDVCs described by the 1D solution; glow discharge in Xe plasma, $p = 30$ Torr, $h = 0.5$ mm. Data from [38].

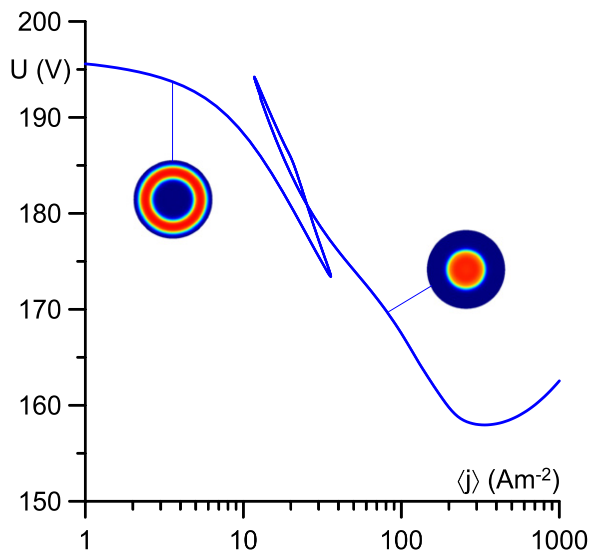

Figure 12. CVC of the CBLD; Xe plasma, $p = 30$ Torr, $R = h = 0.5$ mm. Reprinted from [84].

5.4. Simple situations, complex behavior

Glow discharges and arc–cathode interaction sometimes reveal complex behavior in apparently simple situations, including those where multiple solutions are not of primary concern. One example, which has to do with the effect of lateral surface, has already been illustrated by figure 1 and discussed at the end of section 4.2.

Two further examples are illustrated by figures 11 and 12. Figure 11 depicts the CDVC of a glow discharge in xenon computed in 1D in the framework of a model [38], which is rather detailed and accounts for atomic and molecular ions, electrons, excited atoms, excimers, direct ionization, stepwise ionization, ionization of excimers and non-locality of electron energy. Also shown for comparison is the CDVC computed by means of the basic model, which was used for simulations shown in figures 7(*a*) and 8 and takes into account a single ion species produced via a single effective ionization process and employs the local-field approximation.

Qualitatively there is not much difference between the CDVCs obtained with the basic and detailed models except that the 1D mode computed in the framework of the detailed model, surprisingly, manifests a retrograde behavior in the current density range $200\ \mathrm{A\,m^{-2}} \lesssim j \lesssim 300\ \mathrm{A\,m^{-2}}$; see the *S*-shape manifested by the solid line in figure 11. A similar behavior has been found in the framework of a similar detailed model [84] developed for argon; however, while the retrograde behavior in xenon disappears if stepwise ionization is neglected as seen in figure 11, it does not in argon [38].

The CVC of the discharge in the CBL configuration (planar cathode and a ring-shaped anode) computed in 2D in the framework of the basic model is shown in figure 12 [84]. Surprisingly, it exhibits a loop. (In fact, there is no major difference between this loop and the *S*-shape seen in figure 11: what matters is that in both cases the discharge reveals two turning points, i.e. a retrograde behavior.) The discharge is associated with a pattern comprising a ring spot at the cathode in the range of discharge currents below the loop and a spot at the center in the range of currents above the loop. Neither spot is normal, i.e. the effect of normal current density is absent. The loop is associated with a transition from the pattern with a ring spot to the pattern with a central spot, which occurs as follows: the inner radius of the ring spot decreases and then turns zero (i.e. the ring spot becomes a circle) and the outer radius is somewhat reduced. Note that although methods of numerical simulations of microdischarges are generally well developed, e.g. [85–89], no retrograde behavior of the discharge has apparently been reported. It is unclear whether such behavior could be noticed if a time-dependent solver is employed.

5.5. Observations of spots and patterns on electrodes of gas discharges

Self-organization phenomena in gas discharges are extremely diverse. We merely mention self-organization in the bulk plasma, which ranges from filaments and striations in the discharge column, known for many decades (e.g. [1, 90, 91] and references therein), to exciting new examples such as plasma bullets in atmospheric-pressure plasma jets (e.g. [92, 93]), filamentation in atmospheric-pressure microwave plasmas (e.g. [94, 95]), and liquids and crystals of charged particles in non-ideal plasmas (e.g. [96]).

Note that there is a clear distinction between striations, on the one hand, and filaments and electrode spots, on the other: striations represent a self-organized variation in the direction along the discharge current, while filaments and electrode spots represent a self-organized variation in directions perpendicular to current. Striations are ionization waves governed by ionization kinetics and transport processes, while mechanisms of filaments and electrode spots are quite diverse. A distinction between filaments and electrode spots is clear if the discharge is long enough and possesses a well-pronounced plasma column, which is only weakly dependent on details of current distribution over the electrode surfaces.

Otherwise, this distinction may be rather vague, as mentioned in section 4.4 in connection with the works [26–28, 58–65].

Let us give here a brief summary of observations of spots and patterns on electrodes of gas discharges and relevant first-principles theory and/or modeling wherever exist. Note that many further references can be found in [97].

Observations of modes with multiple spots in dc microdischarges [26–28, 58–65] and their modeling have already been discussed. Note that similar patterns have been observed in CBLD with several circular holes operated in parallel, while self-organization in micro-slit CBLD has a ladder-like structure [28].

Modes with multiple cathode spots have also been observed on cathodes of transient [98] and non-self-sustained dc glow [99–102] discharges. Planar steady-state filaments in the cathode layer of a non-self-sustained glow discharge were computed in [32]. Note that it would be of interest to try to compute patterns of 3D spots on cathodes of non-self-sustained dc discharges by means of the approach based on multiple solutions, reviewed in this paper.

Observations of different modes of current transfer to cathodes of high-pressure arc discharges were reported in [103] for low-current arcs and in [104] for arcs with the current of a few hundred amperes. Many subsequent observations are cited in [6, 37]. A variety of different modes have been observed, the most frequent being the diffuse mode and a constricted, or spot, mode. As indicated in section 4.4, a theory of the diffuse and spot modes has been validated experimentally for low-current arcs. One can also mention the so-called blue-core, or hot-core, mode which occurs on cathodes of low-current free-burning arcs [105–107]. The blue-core mode was observed mostly on cathodes made of thoriated tungsten; with pure tungsten cathodes it was more difficult to obtain [105]. The authors [106] assumed that a convective motion of the gas was necessary for the formation of the blue-core mode. In [108], it was shown that this mode may be explained using the assumption of a temperature-dependent work function, which steeply decreases from 4.55 eV to 3 eV at temperatures above 3000 K due to a thorium ion current.

Diffuse and spot modes also occur on cathodes of vacuum arcs. Spots on cathodes of vacuum arcs possess a complex substructure characterized by several length and time scales; e.g. [109] and references therein. On the largest (macroscopic) scale, such spots are described by models in which processes inside the cathode are treated by means of analytical approximations mentioned in section 5.2 (e.g. [75–78]) or by means of 2D numerical modeling [55, 74]. However, the theory still cannot answer many important questions, such as a scientifically interesting and important for practice question of the effect of the applied magnetic field on the stability of individual spots and, consequently, on current per spot [110, 111]. Also observed on cathodes of vacuum arcs have been patterns of several spots [111–114].

The diffuse, or spotless, mode of current transfer to cathodes of vacuum arcs is known not so well as the spot mode; however, its existence has been firmly established by now [115–121]. Similarly to the diffuse mode on cathodes of ambient-gas arcs, the spotless mode on cathodes of vacuum arcs occurs in cases where the average temperature of the cathode surface is sufficiently high. Values of the cathode surface temperature necessary to this end are typically around 2000 K and can be achieved by placing the (evaporating) cathode in a thermally insulated crucible made of a material for which a vacuum arc would burn at a higher voltage than for the cathode material. This discharge is capable of generating a steady highly ionized plasma containing no microdroplet fraction, which may be useful for applications; e.g. [121]. (Note that this discharge should not be confused with the so-called thermionic vacuum arc discharge [122], which can be ignited under high-vacuum conditions between a heated cathode operating as an electron gun and an evaporating anode placed in a tungsten crucible and heated to a high temperature by the electron beam.) The effort invested by different groups in the experiment and its theoretical analysis has been significant [115–121, 123–126]; however, understanding of the spotless mode on cathodes of vacuum arcs remains elusive. Given that the spotless arc attachment, being in essence a 1D and stationary phenomenon, represents a much simpler object than cathode spots, this state of the art is rather surprising and detrimental not only to potential technological applications of spotless vacuum arc discharges, but to the vacuum arc physics in general. One should hope for further attention to this subject.

Spot patterns are also observed on anodes of dc glow discharges [67, 68, 127–132] and low-current low-pressure arc discharges [133, 134]. Diffuse, constricted and multiple-spot modes are observed on anodes of high-pressure arc discharges [135–140]. Impressive results have been achieved in time-dependent 3D numerical simulations of the multiple-spot mode [34, 35].

Beautiful patterns have been observed on liquid electrodes of glow discharges, both cathodes [141, 142] and anodes [69, 143].

Interesting self-organized dynamic patterns have been observed in an experiment with a dc-driven short glow discharge [144]. A subsequent investigation [145] has revealed that the patterns are accompanied by current pulsations: the discharge current represents a sequence of spikes of duration of the order of a few microseconds. Only one spot per spike exists; however, on a larger time scale it seems that several spots exist simultaneously. The discharge in these experiments is obstructed [145], i.e. operates on the left-hand branch of the Paschen curve, hence the steady-state CDVC monotonically grows (e.g. [1, section 8.3.5] with corrections indicated in [146]). Multiple steady-state solutions do not exist under these conditions, and this is consistent with the fact that the patterns observed in [144] are non-stationary. The question as to why spots appearing in sequence form self-organized patterns remains open.

Negative corona discharges in electron-attaching gases can occur in the form of multiple discrete points, or 'tufts'; e.g. figure 10 and its discussion in section 5.1 and references [147, p 329], [148, 149]. The average current of a tuft corona is steady, but is composed of tiny pulses. On a thin wire electrode, the tufts form a straight line along the wire and are more or less equally spaced. On a thicker wire, the tufts form a pattern resembling a brush. On a point electrode, the tufts form one or more rings around the tip. Given the common physical nature of cathode regions of glow discharges and negative coronas [150], an interesting question is whether there are similarities in the mechanisms of formation of the tuft patterns and patterns of multiple spots on cathodes of dc microdischarges.

A variety of different patterns was observed in dielectric barrier discharges (DBDs); e.g. [97, 151, 152] and references therein. Impressive results have been achieved in their numerical simulation performed both in planar [153–161] and 3D [162, 163] geometry.

Stationary and rotating spot patterns have been observed in a pulsed rf discharge [164]. Well-defined rotating regions of high plasma emissivity and ionization (so-called spokes) forming symmetric patterns have been observed in high-power impulse magnetron sputtering; e.g. [165–168] and references therein.

A wealth of patterns was observed in dc planar glow discharges with a thin interelectrode gap with one of the electrodes being made of a semiconductor material; e.g. [97, 169–171]. Temporal and spatiotemporal patterns in such discharges were simulated in [17, 172, 173]. A 2D steady-state pattern with filaments in such a discharge at very low currents under cryogenic conditions has been successfully computed in [33] and its appearance was attributed to the well-known thermal instability mechanism [174]: an increase in the current density causes an increase in Joule heating, a decrease in the gas density, and therefore an increase in the reduced electric field and the ionization coefficient; see also [175].

A regular filamentary structure develops in a negative polarity nanosecond surface DBD provided that pressure and/or applied voltage are sufficiently high [176]. The authors [176] attributed the filamentation to a thermal mechanism, similar to the one described in the preceding paragraph. As shown in [33, 174, 175], this mechanism can be dominating at very low current densities (in the Townsend regime); however, in the experiments [176] the current densities are estimated to be of the order of the normal current density, i.e. much higher than that in the Townsend regime. Hence, the thermal mechanism can hardly be dominating under the conditions [176]: the cathode sheath mechanism shown in figure 4 is likely to play a role or be dominating.

6. Concluding discussion

6.1. Summary of results in the context of general theory and modeling of gas discharges

Self-consistent theoretical models of dc glow discharges and cathodic part of ambient-gas arc discharges, including the most basic ones, admit multiple solutions existing for the same discharge current. One of these solutions is in the simplest case one-dimensional and describes states with a uniform distribution of current over the cathode surface; this solution is similar to the one given in textbooks. Other solutions are in all the cases multidimensional and describe modes with different self-organized configurations of cathode spots. The existence of multidimensional solutions has been hypothesized long ago; however, they started to be systematically computed only in the last 15 years.

A theory of diffuse and spot modes of current transfer to high-pressure arc cathodes based on the concept of multiple solutions has gone through a detailed experimental validation in low-current arcs and has proved relevant for industrial applications.

Multiple solutions computed in the theory of glow discharges agree with the experiment as well (see section 4.4), although the comparison has been merely qualitative up to now. These solutions allow one to understand and describe in the framework of standard theoretical models, without invoking special mechanisms favoring self-organization, steady-state patterns of multiple spots and 3D and axially symmetric normal cathode spots, as well as their transitions to the abnormal and Townsend discharges.

The mechanism ensuring existence of multiple solutions for both glow and arc cathodes originates in the near-cathode space-charge sheath and is illustrated by figure 4. It follows that basic processes in the near-cathode space-charge sheath are sufficient to produce self-organization. This mechanism, which may be called the cathode sheath instability, is very general and present in all discharges where the near-cathode sheath plays a significant role. It may play a role in the appearance of spots or patterns also on cathodes of ac and pulse discharges.

Understanding of multiple solutions may also be important in apparently simple situations where the issue of multiple solutions seems to be irrelevant.

The existence of multiple solutions describing different modes of current transfer to electrodes is, of course, not a feature specific for dc discharges: in the case of ac and pulse discharges one can think of multiple non-stationary solutions, one of which varies only in the axial direction (is 1D in space) and describes a spotless mode and the others vary also in transversal directions and describe different self-organized modes; see discussion in [161] for the case of DBDs. A feature which is specific for dc discharges is the existence of bifurcations of (steady-state) solutions, and this feature, having been predicted theoretically, was also confirmed experimentally as discussed in section 4.4. This feature represents the basis of the systematic approach to finding multiple solutions in the cases of both dc glow discharges and arc–cathode interaction, described in section 3.2. Such an approach can be used for understanding and modeling spots or patterns also in other dc discharges provided that the current transfer is stationary. Note that it is not sufficient that the discharge is dc-driven and the average discharge current is steady: the discharge current should not be composed of tiny pulses. Let us consider as examples three types of patterns mentioned in section 5.5: (a) patterns observed in the dc planar glow discharge with a thin interelectrode gap with one of the electrodes being made of a semiconductor material, (b) patterns observed in the experiment with a dc-driven short glow discharge [144] and (c) tufts on the negative corona electrodes in electron-attaching gases. One should expect that the approach described in section 3.2 may be applied in case (a) but not in cases (b) and (c).

In contrast to normal spots on glow cathodes and cathode spots in ambient-gas arc discharges, which are single spots, regular patterns of multiple spots, such as those on cathodes of dc glow microdischarges and non-self-sustained dc and transient glow discharges, anodes of glow discharges and ambient-gas arcs, thin glow discharges sandwiched with a semiconductor layer, corona electrodes and DBDs, are clearly self-organization phenomena. Such patterns have been frequently simulated by means of a phenomenological approach in which the distribution of parameters along the electrode surface is assumed to be governed by a reaction–diffusion equation (a diffusion equation with a nonlinear source term) or by a system of coupled reaction–diffusion equations; e.g. [68, 97, 174, 177–181]. For special situations, attempts have been made to derive reaction–diffusion-type equations by means of a two-scale asymptotic technique from basic equations that govern a particular discharge [14, 20, 182, 183]. However, in most situations reaction–diffusion equations for distribution of parameters along the electrode surface are just postulated on the basis of qualitative considerations and features characteristic of nonlinear dissipative systems. On the other hand, there is a growing trend to model patterns in gas discharges from first principles; see, e.g., the above-mentioned first-principles numerical simulation of patterns in DBDs [153–163]. The first-principles simulation of steady-state patterns on glow cathodes, reviewed in this paper, is in line with this trend.

6.2. Effect of normal current density on glow cathodes

Normal spots on glow cathodes and arc cathode spots puzzled researchers for many decades and have generated a number of hypotheses, models and theoretical frameworks that few phenomena in gas discharge physics have. Classic textbooks attributed the effect of normal current density on glow cathodes to 'dispersive forces' acting radially outwards and controlling the emitting cathode area [184, p 224] (this author appealed to the minimum energy principle); instability of the edge of normal spot [1, section 8.4.9]; instability of a discharge operating on the falling section of the CVC with a fixed external voltage source [2, p 549]. A significant development was represented by 2D numerical simulations of normal spots [14, 16, 185–189]. However, the question of how to compute solutions similar to the one which is depicted by the line Nb_1a_1P in figure 7(*a*) and describes the 2D abnormal discharge at high currents, the 2D Townsend discharge at low currents, and the 3D normal discharge with a spot attached to the edge of the cathode at intermediate currents remained unanswered; and the question as to how diffusion of the charged particles to the wall, which is a weak effect, can cause a dramatic difference between the 1D and 2D solutions seen in figure 1 has not even been asked. (Although 2D solutions in a wide current range similar to the one depicted in figure 1 except for the Z-shape connecting the Townsend and subnormal discharges have been computed; see [188, figure 1].)

The approach based on multiple solutions has shown that normal spots on glow cathodes and cathode spots in arc discharges are a part of the general self-organization pattern, thus providing a better understanding and a possibility of systematic computation. In particular, in figure 7(*a*) the effect of normal current density manifests itself as a well-pronounced horizontal section revealed by the CVCs a_1b_1 and a_3Qb_3 and occurs where a large spot coexists with a large current-free region. The line a_1b_1 is qualitatively similar to the line a_1b_1 in figure 6, in agreement with the general theory discussed in section 2.3. The normal voltage is a bit higher than the voltage corresponding to the point of minimum of the CDVC described by the 1D solution, in agreement with [1, section 8.4.9]. Other solutions shown in figure 7(*a*) refer to situations where the spot and/or the current-free region are not large and the effect of normal current density is absent.

According to numerical results [7, figure 2], the current density inside the 2D normal spots is virtually uniform and independent of the discharge current. The same is true for the 3D normal spots. Furthermore, the current densities inside the 2D and 3D normal spots are virtually the same. (The latter can also be deduced from figure 7(*a*): the plateaus revealed by the CVCs a_1b_1 and a_3Qb_3 occur at the same value of U.) All this is consistent with the usual concept of normal current density. However, the normal current density does not coincide with the current density at the minimum of the CDVC, in contrast to what is frequently believed: the former exceeds the latter and the difference under conditions of figure 7(*a*) is about a factor of 2 [7].

The effect of normal current density is a manifestation of coexistence of phases illustrated by figure 5. The coexistence is possible for only one value of the control parameter and this value is determined by the condition of solvability of a problem describing the coexistence; Maxwell's construction (e.g. [21]). In our case, for each set of conditions there is only one value of voltage for which equations of glow discharge (e.g. equations of conservation and transport of charged particles and the Poisson equation) admit a steady-state planar solution describing coexistence of an infinite spot with an infinite current-free region. In other words, steady-state solutions describing coexistence of phases for arbitrary values of voltage do not realize not because they are unstable, but simply because they do not exist. While this reasoning relies on general considerations, it would be of interest to obtain also a direct confirmation by means of a computational investigation of the above-mentioned equations of glow discharge. Note that for arc cathodes this reasoning has been confirmed and an explicit form of Maxwell's construction, governing the normal voltage, derived [9, 20, 22].

6.3. Possible directions of future work

While the theory of diffuse and spot modes on high-pressure arc cathodes based on the concept of multiple solutions has gone through a rather detailed experimental validation, the experimental validation of multiple solutions computed in the theory of glow discharges has been merely qualitative up to now and much further work is required.

While account of non-local electron kinetics is essential for striations in glow discharges (e.g. [91]), main features of filamentation in glow-like discharges have been successfully reproduced in the framework of the most basic self-consistent model which assumes local electron kinetics and takes into account only one ion species and one ionization channel; examples being modeling of pattern formation in DBDs [153–157, 159, 162] and of filamentary plasma arrays in microwave breakdown [190, 191]. Therefore, it is not surprising that main features of spots and patterns observed on cathodes of dc glow discharges have been reproduced in the framework of the above-mentioned basic model. On the other hand, finding multiple solutions in the framework of detailed models of glow discharges, which would also take into account the dependence of the coefficient of electron–ion emission on the electric field strength (e.g. [192, 193]), is of significant interest and further work on this way will hopefully follow.

Steady-state patterns of current transfer predicted by general trends of self-organization theory and computed numerically for dc glow discharges and plasma–cathode interaction in high-pressure arc discharges are similar. However, the experiment reveals significant differences between modes observed in glow microdischarges, on the one hand, and in regular-scale glow discharges and on cathodes of high-pressure arc discharges, on the other. First, patterns with multiple spots have been observed in glow microdischarges but not in regular-scale glows or on cathodes of high-pressure arcs. Second, while axially symmetric current distributions on planar circular cathodes have been observed in glow microdischarges, normal spots in regular-scale glows and cathode spots in high-pressure arc discharges are attached to the edge of the cathode, i.e. are 3D. One should presume that these differences are caused by substantially different properties of the stability of steady-state solutions. While the stability of steady-state solutions describing different modes has been reasonably well understood for cathodes of high-pressure arc discharges, there is considerable amount of work to do as far as glow discharges are concerned.

The vast majority of multiple steady-state solutions found in the theory of glow discharges and plasma–cathode interaction in arc discharges have been computed by means of steady-state solvers. The relevance of the use of steady-state solvers was discussed in section 3.2. On the other hand, it would be helpful if multiple solutions could be computed in some cases also by means of one of the time-dependent solvers which are used by virtually all groups engaged in modeling of gas discharges. Time-dependent solvers are not as demanding in regard to initial condition as steady-state solvers are in regard to initial approximation; besides, time-dependent solvers give information on stability. Therefore, an intelligent combination of steady-state and time-dependent solvers would make a powerful tool.

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References

[1] Raizer Yu P 1991 *Gas Discharge Physics* (Berlin: Springer)
[2] Lieberman M A and Lichtenberg A J 2005 *Principles of Plasma Discharges and Material Processing* 2nd edn (New York: Wiley)
[3] Neumann W 1987 *The Mechanism of the Thermoemitting Arc Cathode* (Berlin: Akademie-Verlag)
[4] Bade W L and Yos J M 1963 Theoretical and experimental investigation of arc plasma-generation technology. Part II, vol 1, A theoretical and experimental study of thermionic arc cathodes *Technical Report* ASD-TDR-62-729 Avco Corporation, Wilmington, MA, USA
[5] Benilov M S 1988 *Sov. Phys.—Tech. Phys.* **33** 1267
[6] Benilov M S 2008 *J. Phys. D: Appl. Phys.* **41** 144001
[7] Almeida P G C, Benilov M S and Faria M J 2010 *Plasma Sources Sci. Technol.* **19** 025019
[8] Almeida P G C, Benilov M S, Cunha M D and Faria M J 2009 *J. Phys. D: Appl. Phys.* **42** 194010
[9] Benilov M S 1998 *Phys. Rev.* E **58** 6480
[10] Melekhin V N and Naumov N Yu 1984 *Sov. Phys.—Tech. Phys.* **29** 888
[11] Melekhin V N, Naumov N Yu and Tkachenko N P 1987 *Sov. Phys.—Tech. Phys.* **32** 274
[12] Phelps A V, Petrović Z Lj and Jelenković B M 1993 *Phys. Rev.* E **47** 2825
[13] Kaganovich I D, Fedotov M A and Tsendin L D 1994 *Tech. Phys.* **39** 241
[14] Kolobov V I and Fiala A 1994 *Phys. Rev.* E **50** 3018
[15] Kudryavtsev A A and Tsendin L D 2002 *Tech. Phys. Lett.* **28** 1036
[16] Arslanbekov R R and Kolobov V I 2003 *J. Phys. D: Appl. Phys.* **36** 2986
[17] Raizer Yu P, Gurevich E L and Mokrov M S 2006 *Tech. Phys.* **51** 185
[18] Benilov M S 2008 *Phys. Rev.* E **77** 036408
[19] Benilov M S 2007 *J. Phys. D: Appl. Phys.* **40** 1376
[20] Benilov M S 1992 *Phys. Rev.* A **45** 5901
[21] Haken H 1978 *Synergetics, An Introduction* (*Springer Series in Synergetics* vol 1) (Berlin: Springer)
[22] Khait V D 1989 *High Temp.* **27** 445
[23] Weizel W and Thouret W 1952 *Z. Phys.* **131** 170
[24] Waymouth J F 1982 *J. Light Vis. Environ.* **6** 53
[25] Fischer E 1987 *Philips J. Res* **42** 58
[26] Schoenbach K H, Moselhy M and Shi W 2004 *Plasma Sources Sci. Technol.* **13** 177
[27] Takano N and Schoenbach K H 2006 *Plasma Sources Sci. Technol.* **15** S109
[28] Zhu W, Takano N, Schoenbach K H, Guru D, McLaren J, Heberlein J, May R and Cooper J R 2007 *J. Phys. D: Appl. Phys.* **40** 3896
[29] Moizhes B Y and Nemchinskii V A 1975 *Sov. Phys.—Tech. Phys.* **20** 757
[30] Islamov R S and Gulamov E N 1998 *IEEE Trans. Plasma Sci.* **26** 7
[31] Surzhikov S T 2008 *Physical and Chemical Kinetics in gas Dynamics (electronic journal)* **7** (in Russian) (www.chemphys.edu.ru/media/files/2008-09-01-029.pdf)
[32] Shveigert V A 1993 *Tech. Phys.* **38** 384
[33] Mokrov M S and Raizer Y P 2011 *J. Phys. D: Appl. Phys.* **44** 425202
[34] Trelles J P 2013 *Plasma Sources Sci. Technol.* **22** 025017
[35] Trelles J P 2014 *Plasma Sources Sci. Technol.* **23** 054002

[36] Almeida P G C, Benilov M S and Faria M J 2011 *J. Phys. D: Appl. Phys.* **44** 415203
[37] Benilov M S, Cunha M D and Faria M J 2009 *J. Phys. D: Appl. Phys.* **42** 145205
[38] Almeida P G C and Benilov M S 2013 *Phys. Plasmas* **20** 101613
[39] Benilov M S and Cunha M D *On-Line Tool for Simulation of Different Modes of Axially Symmetric Current Transfer to Cathodes of High-Pressure Arc Discharges* www.arc_cathode.uma.pt/tool
[40] Faria M J 2011 Stability and bifurcations of modes of current transfer to cathodes of dc gas discharges *PhD Thesis* Universidade da Madeira (http://fisica.uma.pt/public/PhDs/Thesis_MJ.pdf)
[41] Benilov M S and Cunha M D 2003 *Phys. Rev.* E **68** 056407
[42] Dabringhausen L, Langenscheidt O, Lichtenberg S, Redwitz M and Mentel J 2005 *J. Phys. D: Appl. Phys.* **38** 3128
[43] Benilov M S, Carpaij M and Cunha M D 2006 *J. Phys. D: Appl. Phys.* **39** 2124
[44] Lenef A 2006 *Proc. Comsol Users Conference 2006 (Boston, MA, 22–24 October 2006)* ed J Hiller (Boston, MA: COMSOL) pp 125–30, ISBN 0-9766792-2-1
[45] Bergner A, Westermeier M, Ruhrmann C, Awakowicz P and Mentel J 2011 *J. Phys. D: Appl. Phys.* **44** 505203
[46] Benilov M S and Cunha M D 2013 *J. Appl. Phys.* **113** 063301
[47] Benilov M S and Cunha M D 2003 *J. Phys. D: Appl. Phys.* **36** 603
[48] Bötticher R and Bötticher W 2000 *J. Phys. D: Appl. Phys.* **33** 367
[49] Bötticher R and Bötticher W 2001 *J. Phys. D: Appl. Phys.* **34** 1110
[50] Bötticher R, Graser W and Kloss A 2004 *J. Phys. D: Appl. Phys.* **37** 55
[51] Bötticher R and Kettlitz M 2006 *J. Phys. D: Appl. Phys.* **39** 2715
[52] Benilov M S and Faria M J 2007 *J. Phys. D: Appl. Phys.* **40** 5083
[53] Almeida P G C, Benilov M S and Faria M J 2011 *IEEE Trans. Plasma Sci.* **39** 2190
[54] Almeida P G C, Benilov M S and Santos D F N 2014 arXiv:1406.4394
[55] Benilov M S, Cunha M D, Hartmann W and Wenzel N 2014 *Plasma Sources Sci. Technol.* **23** 054007
[56] Almeida P G C, Benilov M S, Cunha M D and Gomes J G L 2013 *Plasma Sources Sci. Technol.* **22** 012002
[57] Schmidt M, Schneidenbach H and Kettlitz M 2013 *J. Phys. D: Appl. Phys.* **46** 435202
[58] Moselhy M and Schoenbach K H 2004 *J. Appl. Phys.* **95** 1642
[59] Korolev Yu D and Schoenbach K H 2005 *Proc. 27th ICPIG (Eindhoven, the Netherlands, July 2005)* ed E M van Veldhuizen (Eindhoven: Eindhoven University of Technology)
[60] Takano N and Schoenbach K H 2006 *Abstracts of the 2006 IEEE Int. Conf. on Plasma Science (IEEE, Traverse City, MI)* p 247
[61] Lee B-J, Rahaman H, Frank K, Mares L and Biborosch D-L 2007 *Proc. 28th ICPIG (Prague, July 2007)* ed J Schmidt *et al* (Prague: Institute of Plasma Physics AS CR) pp 900–2, ISBN 978-80-87026-01-4
[62] Lee B-J, Biborosch D-L, Frank K and Mares L 2008 *J. Optoelectron. Adv. Mater.* **10** 1972
[63] Schoenbach K H and Zhu W 2012 *IEEE J. Quantum Electron.* **48** 768
[64] Zhu W, Niraula P, Almeida P G C, Benilov M S and Santos D F N 2014 *Plasma Sources Sci. Technol.* **23** 054012
[65] Zhu W and Niraula P 2014 *Plasma Sources Sci. Technol.* **23** 054011
[66] Gurevich E L, Zanin A L, Moskalenko A S and Purwins H-G 2003 *Phys. Rev. Lett.* **91** 154501
[67] Thomas C H and Duffendack O S 1930 *Phys. Rev.* **35** 72
[68] Müller K G 1988 *Phys. Rev.* A **37** 4836
[69] Shirai N, Uchida S and Tochikubo F 2014 *Plasma Sources Sci. Technol.* **23** 054010
[70] Egli W, Riccius O, Kogelschatz U, Gruber R and Merazzi S 1994 *Proc. 6th Joint EPS-APS Int. Conf. on Physics Computing (Lugano, Switzerland)* ed R Gruber and M Tomassini, p 535
[71] Egli W and Kogelschatz U 1995 *Contributed Papers of 22nd Int. Conf. on Phenomena in Ionized Gases (Hoboken, NJ, 1995)* vol 1 ed K H Becher *et al* p 119
[72] Benilov M S 1997 *Phys. Lett.* A **228** 182
[73] Egli W and Kogelschatz U 1995 *Helv. Phys. Acta* **68** 203
[74] Benilov M S, Cunha M D, Hartmann W, Kosse S, Lawall A and Wenzel N 2013 *IEEE Trans. Plasma Sci.* **41** 1950
[75] Beilis I I 2010 *Appl. Phys. Lett.* **97** 121501
[76] Beilis I I 2011 *IEEE Trans. Plasma Sci.* **39** 1277
[77] Beilis I I 2013 *IEEE Trans. Plasma Sci.* **41** 1979
[78] Almeida N A, Benilov M S, Benilova L G, Hartmann W and Wenzel N 2013 *IEEE Trans. Plasma Sci.* **41** 1938
[79] Benilov M S 1998 *Phys. Scr.* **58** 383
[80] Benilov M S 2004 *IEEE Trans. Plasma Sci.* **32** 249
[81] Steenbeck M 1932 *Phys. Z.* **33** 809
[82] Ecker G 1980 *Vacuum Arcs: Theory and Application* ed J M Lafferty (New York: Wiley) pp 228–320
[83] Benilov M S and Naidis G V 2010 *J. Phys. D: Appl. Phys.* **43** 175204
[84] Almeida P G C 2011 Investigation of modes of current transfer in dc glow and arc discharges *PhD Thesis* Universidade da Madeira (http://fisica.uma.pt/public/PhDs/Thesis_Pedro.pdf)
[85] Boeuf J P, Pitchford L C and Schoenbach K H 2005 *Appl. Phys. Lett.* **86** 071501
[86] Muñoz-Serrano E, Hagelaar G, Callegari Th, Boeuf J P and Pitchford L C 2006 *Plasma Phys. Control. Fusion* **48** B391
[87] Makasheva K, Muñoz-Serrano E, Hagelaar G, Boeuf J P and Pitchford L C 2007 *Plasma Phys. Control. Fusion* **49** B233
[88] Deconinck T and Raja L L 2009 *Plasma Process. Polym.* **6** 335
[89] Zhang X, Wang X, Liu F and Lu Y 2009 *IEEE Trans. Plasma Sci.* **37** 2055
[90] Velikhov E P, Kovalev E P and Rakhimov A T 1987 *Physical Phenomena in a Gas Discharge Plasma* (Moscow: Nauka) (in Russian)
[91] Kolobov V I 2006 *J. Phys. D: Appl. Phys.* **39** R487
[92] Teschke M, Kedzierski J, Finantu-Dinu E G, Korzec D and Engemann J 2005 *IEEE Trans. Plasma Sci.* **33** 310
[93] Lu X, Naidis G, Laroussi M and Ostrikov K 2014 *Phys. Rep.* **540** 123
[94] Hidaka Y, Choi E M, .Mastovsky I, Shapiro M A, Sirigiri J R and Temkin R J 2008 *Phys. Rev. Lett.* **100** 035003
[95] Cardoso R P, Belmonte T, Noël C, Kosior F and Henrion G 2009 *J. Appl. Phys.* **105** 093306
[96] Bonitz M, Kählert H, Ludwig P, Ott T, Thomsen H, Löwen H, Block D, Melzer A and Piel A 2012 *ESCAMPIG XXI (Viana do Castelo, Portugal, 10–14 July 2012)*
[97] Purwins H-G, Bödeker H U and Amiranashvili S 2010 *Adv. Phys.* **59** 485
[98] Farish O and Tedford D J 1966 *Br. J. Appl. Phys.* **17** 965
[99] Korolev Yu D, Rabotkin V G and Filonov A G 1979 *High Temp.* **17** 181
[100] Genkin S A, Korolev Yu D and Khuzeev A P 1982 *Sov. Phys.—Tech. Phys.* **27** 560
[101] Bystrov S A, Lushchikova A M, Mazalov D A, Pal A F, Starostin A N, Taran M D, Taran T V and Filippov A V 1994 *J. Phys. D: Appl. Phys.* **27** 273

[102] Arkhipenko V, Kirillov A, Callegari T, Safronau Y and Simonchik L 2009 *IEEE Trans. Plasma Sci.* **37** 740
[103] Thouret W, Weizel W and Günther P 1951 *Z. Phys.* **130** 621
[104] Olsen H N 1963 *J. Quant. Spectrosc. Radiat. Transfer* **3** 305
[105] Reiche J, Könemann F, Mende W and Kock M 2001 *J. Phys. D: Appl. Phys.* **34** 3177
[106] Könemann F, Kühn G, Reiche J and Kock M 2004 *J. Phys. D: Appl. Phys.* **37** 171
[107] Kühn G and Kock M 2007 *Phys. Rev.* E **75** 016406
[108] Bergner A, Scharf F H, Kühn G, Ruhrmann C, Hoebing T, Awakowicz P and Mentel J 2014 *Plasma Sources Sci. Technol.* **23** 054005
[109] Jüttner B 2001 *J. Phys. D: Appl. Phys.* **34** R103
[110] Chaly A M, Logatchev A A, Zabello K K and Shkol'nik S M 2007 *IEEE Trans. Plasma Sci.* **35** 946
[111] Chaly A M, Barinov Yu A, Minaev V S, Myatovich S U, Zabello K K and Shkol'nik S M 2013 *IEEE Trans. Plasma Sci.* **41** 1917
[112] Kesaev I G 1963 *Sov. Phys.—Tech. Phys.* **8** 447
[113] Sherman J C, Webster R, Jenkins J E and Holmes R 1975 *J. Phys. D: Appl. Phys.* **8** 696
[114] Chaly A M, Logatchev A A, Zabello K K and Shkol'nik S M 2003 *IEEE Trans. Plasma Sci.* **31** 884
[115] Vasin A I, Dorodnov A M and Petrosov V A 1979 *Sov. Tech. Phys. Lett.* **5** 634
[116] Paranin S N, Polishchuk V P, Sychev P E, Shabashov V I and Yartsev I M 1986 *High Temp.* **24** 307
[117] Polishchuk V P, Sychev P E, Shabashov V I and Yartsev I M 1986 *Sov. Phys.—Tech. Phys.* **31** 1336
[118] Bronin S Ya, Polishchuk V P, Sychev P E, Shabashov V I and Yartsev I M 1986 *Investigation of the Near-Cathode Phenomena in a Stationary Arc with a Diffuse Cathode Anchoring* Preprint No 2-199 Institute for High Temperatures of the USSR Academy of Sciences, Moscow
[119] Bronin S Ya, Polishchuk V P, Sychev P E, Shabashov V I and Yartsev I M 1993 *High Temp.* **31** 33
[120] Batenin V M, Klimovskii I I, Polishchuk V P and Sinel'shchikov V A 2003 *High Temp.* **41** 586
[121] Amirov R K, Vorona N A, Gavrikov A V, Lizyakin G D, Polischuk V P, Samoylov I S, Smirnov V P, Usmanov R A and Yartsev I M *Proc. Conf. on Physics of Low-Temperature Plasma FNTP 2014 (Kazan, Russia, 20–23 May 2014)* vol 1 (Kazan: KNITU) pp 130–4
[122] Akan T, Demirkol S, Ekem N, Pat S and Musa G 2007 *Plasma Sci. Technol.* **9** 280
[123] Bolotov A V, Kozyrev A V and Korolev Yu D 1993 *Fiz. Plasmy* **19** 709 (in Russian)
[124] Bolotov A, Kozyrev A and Korolev Yu 1995 *IEEE Trans. Plasma Sci.* **23** 884
[125] Benilov M S 1997 *Progress in Plasma Processing of Materials* ed P Fauchais (New York: Begell House) pp 815–21
[126] Benilov M S and Benilova L G 2014 Analyzing spotless mode of current transfer to cathodes of metal-vapor arcs *Proc. ISDEIV 2014 (Mumbai, India, 28 September–3 October 2014)* submitted
[127] Rubens S M and Henderson J E 1940 *Phys. Rev.* **58** 446
[128] Emeleus K G 1982 *Int. J. Electron.* **52** 407
[129] Maszl Ch, Laimer J and Störi H 2011 *IEEE Trans. Plasma Sci.* **39** 2118
[130] Arkhipenko V I, Safronau V I, Simonchik L V and Tsuprik I M 2012 *ESCAMPIG XXI (Viana do Castelo, Portugal, 10-14 July 2012)*
[131] Sivoš J, Škoro N, Marić D, Malović G and Petrović Z Lj 2012 *Contributed Papers 26th Summer School Int. Symp. on Physics of Ionized Gases (Zrenjanin, Serbia, 27–31 August 2012)* ed M Kuraica and Z Mijatović (Novi Sad: University of Novi Sad) pp 273–6
[132] Arkhipenko V I, Callegari T, Safronau Y A, Simonchik L V and Tsuprik I M 2013 *Plasma Sources Sci. Technol.* **22** 045003
[133] Güntherschulze V A, Bär W and Betz H 1938 *Z. Phys.* A **109** 293
[134] Klyarfel'd B N and Neretina N A 1960 *Sov. Phys.—Tech. Phys.* **5** 169
[135] Baksht F G, Kostin A A, Mitrofanov N K and Shkol'nik S M 1995 *Proc. Conf. on Physics of Low-Temperature Plasma (Petrozavodsk, Russia, June 1995)* (Petrozavodsk: PGU) pp 191–3 (in Russian)
[136] Baksht F G, Dyuzhev G A, Mitrofanov N K and Shkol'nik S M 1997 *Tech. Phys.* **42** 35
[137] Shkol'nik S M 2000 *Encyclopaedia of Low-Temperature Plasmas* vol 2, ed V E Fortov (Moscow: Nauka) pp 147–65 (in Russian)
[138] Yang G and Heberlein J 2007 *Plasma Sources Sci. Technol.* **16** 529
[139] Heberlein J, Mentel J and Pfender E 2010 *J. Phys. D: Appl. Phys.* **43** 023001
[140] Shkol'nik S M 2011 *Plasma Sources Sci. Technol.* **20** 013001
[141] Gaysin F M and Son E E 1989 *Electrophysical Processes in the Discharges of Liquid and Solid Electrodes* (Sverdlovsk: Urals State University) (in Russian)
[142] Gaysin F M, Son E E and Shakirov V D 1990 *Initialization and Development of the Volume Discharge with Liquid Electrodes* (Moscow: Polytechnical Institute) (in Russian)
[143] Verreycken T, Bruggeman P and Leys C 2009 *J. Appl. Phys.* **105** 083312
[144] Nasuno S 2003 *Chaos* **13** 1010
[145] Gurevich E L, Amiranashvili Sh and Purwins H-G 2005 *J. Phys. D: Appl. Phys.* **38** 1029
[146] Raizer Yu P, Ebert U and Šijačić D D 2004 *Phys. Rev.* E **70** 017401
[147] Loeb L B 1965 *Electrical Coronas* (Berkeley, CA: University of California Press)
[148] Chang J-S, Lawless P A and Yamamoto T 1991 *IEEE Trans. Plasma Sci.* **19** 1152
[149] Gutsol A F and Pyle W R 2014 *Plasma Sources Sci. Technol.* **23** 054015
[150] Akishev Y, Grushin M, Kochetov I, Karal'nik V, Napartovich A and Trushkin N 2005 *Plasma Sources Sci. Technol.* **14** S18
[151] Kogelschatz U 2002 *IEEE Trans. Plasma Sci.* **30** 1400
[152] Kogelschatz U 2010 *J. Phys.: Conf. Ser.* **257** 012015
[153] Muller L, Punset C, Ammelt E, Purwins H-G and Boeuf J-P 1999 *IEEE Trans. Plasma Sci.* **27** 20
[154] Brauer I, Punset C, Purwins H-G and Boeuf J 1999 *J. Appl. Phys.* **85** 7569
[155] Shang W-L and Wang D-Z 2007 *Chin. Phys. Lett.* **24** 1992
[156] Bernecker B, Callegari T, Blanco S, Fournier R and Boeuf J P 2009 *Eur. Phys. J. Appl. Phys.* **47** 22808
[157] Bernecker B, Callegari T and Boeuf J P 2010 *Proc. 12th HAKONE (Bratislava, Slovakia, September 2010)* ed J Országh *et al* (Bratislava: Society for Plasma Research and Applications, Library and Publishing Centre CU) pp 19–30, ISBN 978-80-89186-72-3
[158] Shkurenkov I A, Mankelevich Y A and Rakhimova T V 2011 *Eur. Phys. J.* D **61** 95
[159] Bernecker B, Callegari T and Boeuf J P 2011 *J. Phys. D: Appl. Phys.* **44** 262002
[160] Shkurenkov I A, Mankelevich Yu A and Rakhimova T V 2013 *Plasma Sources Sci. Technol.* **22** 015021
[161] Callegari T, Bernecker B and Boeuf J P 2014 *Plasma Sources Sci. Technol.* **23** 054003
[162] Stollenwerk L, Amiranashvili Sh, Boeuf J-P and Purwins H-G 2006 *Phys. Rev. Lett.* **96** 255001
[163] Bhoj A N and Kolobov V I 2011 *IEEE Trans. Plasma Sci.* **39** 2152

[164] Voronov M, Hoffmann V, Steingrobe T, Buscher W, Engelhard C, Storey A, Ray S J and Hieftje G M 2014 *Plasma Sources Sci. Technol.* **23** 054009
[165] Kozyrev A V, Sochugov N S S, Oskomov K V, Zakharov A N and Odivanova A N 2011 *Plasma Phys. Rep.* **37** 621
[166] Anders A, Ni P and Rauch A 2012 *J. Appl. Phys.* **111** 053304
[167] Anders A 2012 *Appl. Phys. Lett.* **100** 224104
[168] de los Arcos T, Schröder R, Gonzalvo Y A, Schulz von der Gathen V and Winter J 2014 *Plasma Sources Sci. Technol.* **23** 054008
[169] Strümpel C, Purwins H-G and Astrov Yu A 2001 *Phys. Rev.* E **63** 026409
[170] Gurevich E L, Astrov Yu A and Purwins H-G 2005 *J. Phys. D: Appl. Phys.* **38** 468
[171] Astrov Y A and Purwins H-G 2006 *Phys. Lett.* A **358** 404
[172] Šijačić D D, Ebert U and Rafatov I 2005 *Phys. Rev.* E **71** 066402
[173] Rafatov I R, Šijačić D D and Ebert U 2007 *Phys. Rev.* E **76** 036206
[174] Raizer Yu P and Mokrov M S 2010 *J. Phys. D: Appl. Phys.* **43** 255204
[175] Raizer Yu P and Mokrov M S 2013 *Phys. Plasmas* **20** 101604
[176] Stepanyan S A, Starikovskiy A, Popov N A and Starikovskaia S M 2014 *Plasma Sources Sci. Technol.* **23** 045003
[177] Kozyrev A V and Korolev Yu D 1983 *Sov. J. Plasma Phys.* **9** 503
[178] Mark P and Müller K G 1991 *J. Appl. Phys.* **70** 6694
[179] Bödeker H U, Röttger M C, Liehr A W, Frank T D, Friedrich R and Purwins H-G 2003 *Phys. Rev.* E **67** 056220
[180] Astrov Yu A and Logvin Yu A 1997 *Phys. Rev. Lett.* **79** 2983
[181] Gurevich S V, Bödeker H U, Moskalenko A S, Liehr A W and Purwins H-G 2004 *Physica* D **199** 115
[182] Islamov R S 2001 *Phys. Rev.* E **64** 046405
[183] Amiranashvili Sh, Gurevich S V and Purwins H-G 2005 *Phys. Rev.* E **71** 066404
[184] von Engel A 1993 *Ionized Gases* (*American Vacuum Society Classics*) (Woodbury, NY: AIP)
[185] Gladush G G and Samokhin A A 1981 *J. Appl. Mech. Tech. Phys.* **22** 608
[186] Raizer Yu P and Surgikov S T 1987 *Sov. Tech. Phys. Lett.* **13** 186
[187] Raizer Yu P and Surgikov S T 1988 *High Temp.* **26** 304
[188] Fiala A, Pitchford L C and Boeuf J P 1994 *Phys. Rev.* E **49** 5607
[189] Surzhikov S T 2005 *High Temp.* **43** 825
[190] Boeuf J P, Chaudhury B and Zhu G Q 2010 *Phys. Rev. Lett.* **104** 015002
[191] Chaudhury B, Boeuf J P and Zhu G Q 2010 *Phys. Plasmas* **17** 123505
[192] Azarov A V, Mitko S V and Ochkin V N 2002 *Bull. Lebedev Phys. Inst.* **No 4** 11
[193] Azarov A V and Ochkin V N 2004 *J. Russ. Laser Res.* **25** 138