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Modelling 3D spot modes on cathodes of high-pressure arc discharges

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Steady-state current transfer from arc plasmas to axially symmetric cathodes is treated in the framework of the model of nonlinear surface heating. Three-dimensional spot modes which branch off from the diffuse mode have been calculated in the whole domain of their existence.

1. Introduction

Current transfer from high-arc pressure plasmas to thermionic cathodes may occur in a diffuse mode, when the current is distributed over the front surface of the cathode in a more or less uniform way, or in a spot mode, when most of the current is localized in one or more small areas (cathode spots). In a certain arc current range, either of the modes can occur.

It was suggested some years ago [1] that an adequate theoretical description of multiple modes of current transfer to thermionic cathodes does not necessarily involve essentially different physical mechanisms but is rather a mathematical question of finding non-unique solutions: an adequate theoretical model of current transfer to hot cathodes must in some cases allow different steady-state solutions to exist for the same conditions, which describe different modes of current transfer. It was shown that such multiple solutions exist in the framework the model of nonlinear surface heating and a general pattern of solutions describing various modes was suggested on the basis of bifurcation analysis and general considerations.

In the subsequent years, the model of nonlinear surface heating was validated by an extensive comparison with the experiment [2, 3] and has become a widely accepted tool of modelling of interaction of high-pressure arc plasmas with thermionic cathodes. At present, steady-state axially symmetric modes on axially symmetric cathodes have been (numerically) studied in detail and are understood relatively well (e.g., [2, 4, 5]); this applies to both diffuse and axially symmetric spot modes. Numerical results on three-dimensional (3D) spot modes have started to appear only recently [6–8]. In [6], bifurcation points have been calculated in which 3D steady-state spot modes on axially symmetric cathodes branch off from axially symmetric modes. It was shown that 3D spot modes can branch off from both the diffuse mode and axially symmetric spot modes. 3D transient spots on a cylindrical cathode were simulated in [7]. In [8], results of simulations are given of a 3D steady-state spot attached to a rounded edge of a cylindrical cathode.

The physics of 3D spots should be similar to the physics of axially symmetric spots, which has been studied in detail [5]. The importance of computational aspects has considerably decreased in recent years due to appearance of powerful computers and advanced software. There is, however, an aspect in modelling of 3D spot modes which is still of primary interest, namely, finding the general pattern of different modes. In particular, this pattern is needed in order to permit identification of a particular mode of current transfer observed in simulations or in the experiment. It is also critical for analysis of stability. Note that available works diverge as far as this pattern is concerned: a 3D spot mode reported in [8] just terminates instead of turning back or joining an axially symmetric solution, which does not fit in the pattern [1, 6] and is not a behavior typical of multiple solutions.

Establishing the pattern of different 3D modes of current transfer to thermionic cathodes represents the goal of this work.

2. The model

The model of nonlinear surface heating used in this work is described in detail elsewhere [2] and can be summarized as follows. A steady-state temperature distribution in the body of a cathode is considered. Joule heat production in the body of cathode is neglected. The base of the cathode is maintained at a fixed temperature T_c by external cooling and the rest of the cathode surface is in contact with the plasma or the cold gas and is heated or, respectively, cooled. Mathematically, the problem amounts to solving the thermal-conduction equation

$$\nabla \cdot (\kappa \nabla T) = 0$$

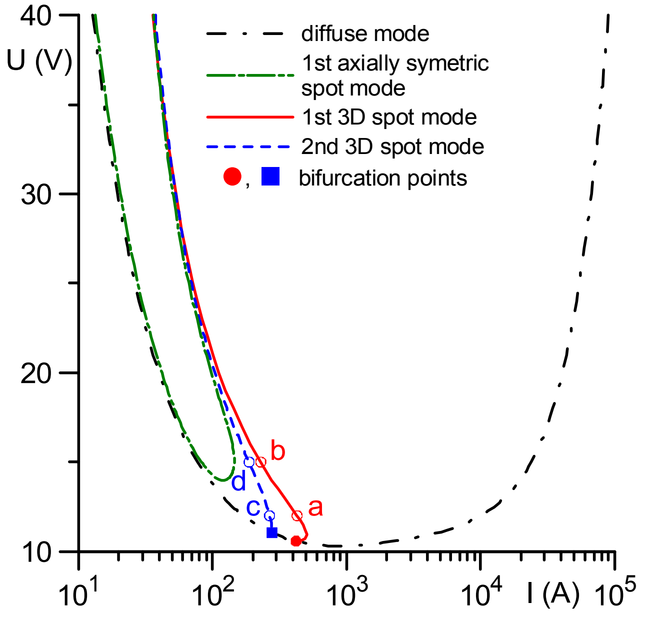

Figure 1: Current-voltage characteristics of various modes of current transfer.

with the boundary condition $T = T_c$ at the base and with the nonlinear boundary condition

$$\kappa \frac{\partial T}{\partial n} = q\left(T_w, U\right)$$

at the rest of the cathode surface. Here κ is thermal conductivity of the cathode material, n is a direction locally orthogonal to the cathode surface and directed outside the cathode and $q = q\left(T_w, U\right)$ is a given function of the cathode surface temperature T_w and of the near-cathode voltage drop U which describes the density of the energy flux to the part of the cathode surface that is in contact with the arc plasma and with the cold gas. This function, as well as the function $j = j\left(T_w, U\right)$ describing the density of electric current to the cathode surface are calculated as described in [9] with modifications introduced in [2,6].

In this work, the above-stated problem was solved numerically by means of the commercial software FEMLAB.

3. Numerical results and discussion

Numerical results given in this work refer to a tungsten cathode in the form of a right circular cylinder (a rod) operating in the atmospheric-pressure argon plasma. Current-voltage characteristics (CVC's) of various modes of current transfer to the cathode of a radius $R = 2\,\mathrm{mm}$ and a height $h = 10\,\mathrm{mm}$ are shown in figure 1.

CVC's of the diffuse mode and of the first axially symmetric spot mode shown in figure 1 coincide with those calculated in [5] and are not discussed here; we note only that the CVC of the diffuse mode has two branches, one falling and the other rising, separated by a point of minimum, while the CVC of the first axially symmetric spot mode has two branches, a low-voltage one and a high-voltage one, separated by a turning point. Note that the diffuse mode and the first axially symmetric spot mode have been calculated in this work by means of the finite-element FEMLAB software and in [5] by means of a Fortran code implementing an iterative approach based on a finite-difference numerical scheme. It should be stressed that results given by the two codes coincide to a very high accuracy, which attests to accurate operation of both codes.

Also shown in figure 1 are bifurcation points in which 3D solutions branch off from the diffuse mode solution. These points have been calculated as described in [6] and also are not discussed here. We note only that the azimuthal dependence of 3D solutions in the vicinity of bifurcation points is described by the factor $\cos k\phi$, where $k = 1, 2, 3, \ldots$ and ϕ is the azimuthal angle; bifurcation point depicted by full circle is associated with $k = 1$ and the one depicted by square is associated with $k = 2$. Note also that, strictly speaking, what branches off at each bifurcation point is a family of solutions rather than a single solution; however, these solutions are identical to the accuracy of a rotation and can therefore be considered as a single solution with an arbitrary azimuthal position of the spot system.

In contrast to the axially symmetric modes and bifurcation points, 3D spot modes shown in figure 1 have not been calculated previously. Each one of these modes branches off from the diffuse mode, in accord to the general pattern suggested in [1,6]. The bifurcation points at which this branching occurs coincide with those predicted by the bifurcation theory [6]. Note that first-harmonic 3D solution (i.e., 3D solution branching off in the bifurcation point associated with $k = 1$) is even with respect to ϕ, therefore the calculation domain used in simulations of this solution comprises only half of the cathode. Similarly, second-harmonic 3D solution (i.e., 3D solution branching off in the bifurcation point associated with $k = 2$) has two planes of symmetry and was calculated in a quarter of the cathode.

Both 3D modes bifurcate from diffuse mode into the range $I > I_b$, where I_b is the current corresponding to the bifurcation point. However, in the end all spot modes tend into the region of small currents and high voltages, i.e., approach the axis of voltages. General considerations explaining this behaviour have been given in [1].

Sometimes CVC's of different modes are close between themselves. This, however, does not mean that thermal regimes of the corresponding modes are also close. It is important to emphasize that variations in the vicinity of the bifurcation points of the temperatures corresponding to the 3D modes are quite smooth, which once again confirms the conclusion that each 3D mode branches off from diffuse mode and the branching occurs at a bifurcation point predicted by the theory [6].

Evolution of temperature distributions along 3D spot modes is shown in figures 2 and 3. The first state shown for each mode corresponds to the bifurcation point at which this mode branches off from the diffuse mode, the second and third states are indicated in figures 1 by open circles, and the fourth state for each mode corresponds to $U = 40\,\mathrm{V}$. The first state shown in figure 2, being a bifurcation point, belongs to the diffuse mode and is characterized by a nearly constant temperature of the front surface of the cathode. The second state, being still close to the bifurcation point, is characterized by smooth 3D perturbations. As the distance from the bifurcation point grows (the third state), the perturbations evolve into a well defined spot at the edge of the front surface of the cathode. As the distance from the bifurcation point grows further and the current-voltage characteristic tends into the region of small currents and high voltages (the fourth state), the spot shrinks and becomes brighter. Evolution of the temperature distribution along the second-harmonic 3D spot mode branching off from the diffuse mode (figure 3) is similar, the difference being that this mode is characterized by two spots (rather than one) at the edge positioned opposite each other.

Since the spots shrink when the CVC's tend into the region of small currents and high voltages, it is possible that in this region the interaction of each spot with distant parts of the cathode and with other spots (if they exist) becomes less important and the spot starts to behave as solitary.

According to the general theory [5], the temperature inside a solitary spot is close to the limiting temperature T_2, which is the value of the surface temperature starting from which thermionic cooling exceeds the combined heating by the ions and plasma electrons and the function q turns negative. In this connection, the maximum temperature of the cathode surface for different modes and the corresponding limiting temperature T_2 are shown in figure 4. Note that the maximum temperature occurs at the edge of the front surface at $\phi = 0$; if the distance from the bifurcation point is not too small and spot(s) are well defined already, this temperature may be interpreted as the maximum temperature inside the edge spot(s). Since $T_2 = T_2\,(U)$, values of T_2 shown in figure 4 for each mode were calculated with the use of the CVC, $U = U\,(I)$, of the respective mode: $T_2 = T_2\,[U\,(I)]$.

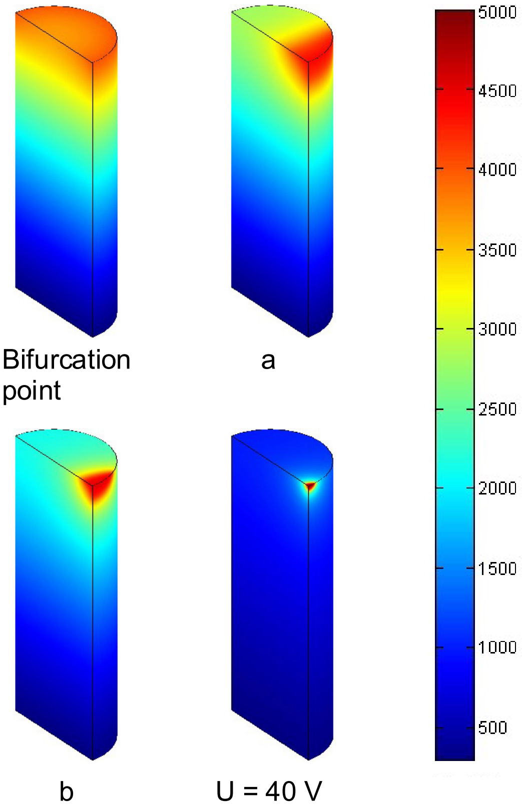

Figure 2: Evolution of temperature distributions along the 1st 3D spot mode. Temperature in the bar is given in Kelvin.

The maximum temperature attained by the cathode in each mode remains below the upper limit of the cathode temperature as it should. As the current decreases, the difference between these two temperatures becomes very small. This is an indication that the spots indeed start behaving like solitary as the current decreases. However, even at the minimum value of the current calculated, these spots still cannot be considered as fully developed solitary spots since they still continue to be influenced by the cathode geometry.

4. Conclusions

A numerical investigation of steady-state 3D modes on a cylindrical cathode is reported. Results are

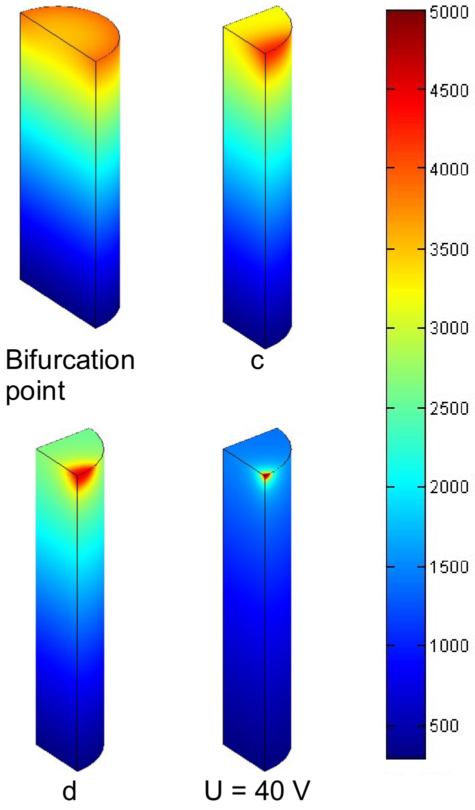

Figure 3: Evolution of temperature distributions along the 2nd 3D spot mode. Temperature in the bar is given in Kelvin.

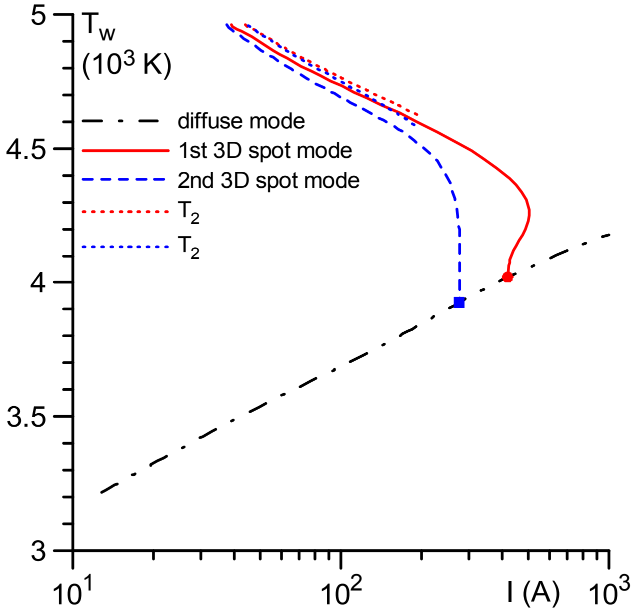

Figure 4: Maximum temperature of the edge of the front surface of the cathode in different modes of current transfer.

given on the first and second 3D modes branching off from the diffuse mode. Each mode is calculated in the whole domain of its existence, from the bifurcation point in which it branches off from the diffuse mode down to very low currents. One can clearly observe the evolution from smooth 3D perturbations in the vicinity of bifurcation points into well defined edge spots at some distance from bifurcation point. At very low currents, the corresponding near-cathode voltage drop tends to infinity and each hot spot shrinks and its behaviour approaches that of a solitary spot.

The present numerical results confirm the general pattern of CVC's of various modes suggested in [1,6] on the basis of bifurcation analysis and general considerations. It is shown, in particular, that the current range in which 3D spot modes exist is limited from above; a result important for understanding the diffuse-spot transition.

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