Universidade da Madeira

Departamento de Física

THEORY AND MODELLING OF CURRENT CONSTRICTION ON CATHODES OF DC DISCHARGES

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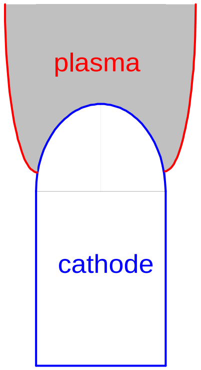
ESCAMPIG XVIII

Lecce, Italy, July 15, 2006

Acknowledgements

- Projects from Fundação para a Ciência e a Tecnologia, EU, Asea Brown Boveri, General Electric Co, …
- M. D. Cunha, G. V. Naidis, N. A. Almeida, M. Carpaij, M. J. Faria, ...

Spots on cathodes in high-pressure arcs



The current is distributed over the front surface of the cathode in a more or less uniform way; the **diffuse mode**.

The current is localized in a region occupying a small fraction of the surface (cathode spot); a **spot mode**.

Side-on observation of a cathode of an arc discharge in argon.
W, R = 0.75 mm, p = 4.5 bar, I = 2.5 A. From S. Lichtenberg *et al* 2002.

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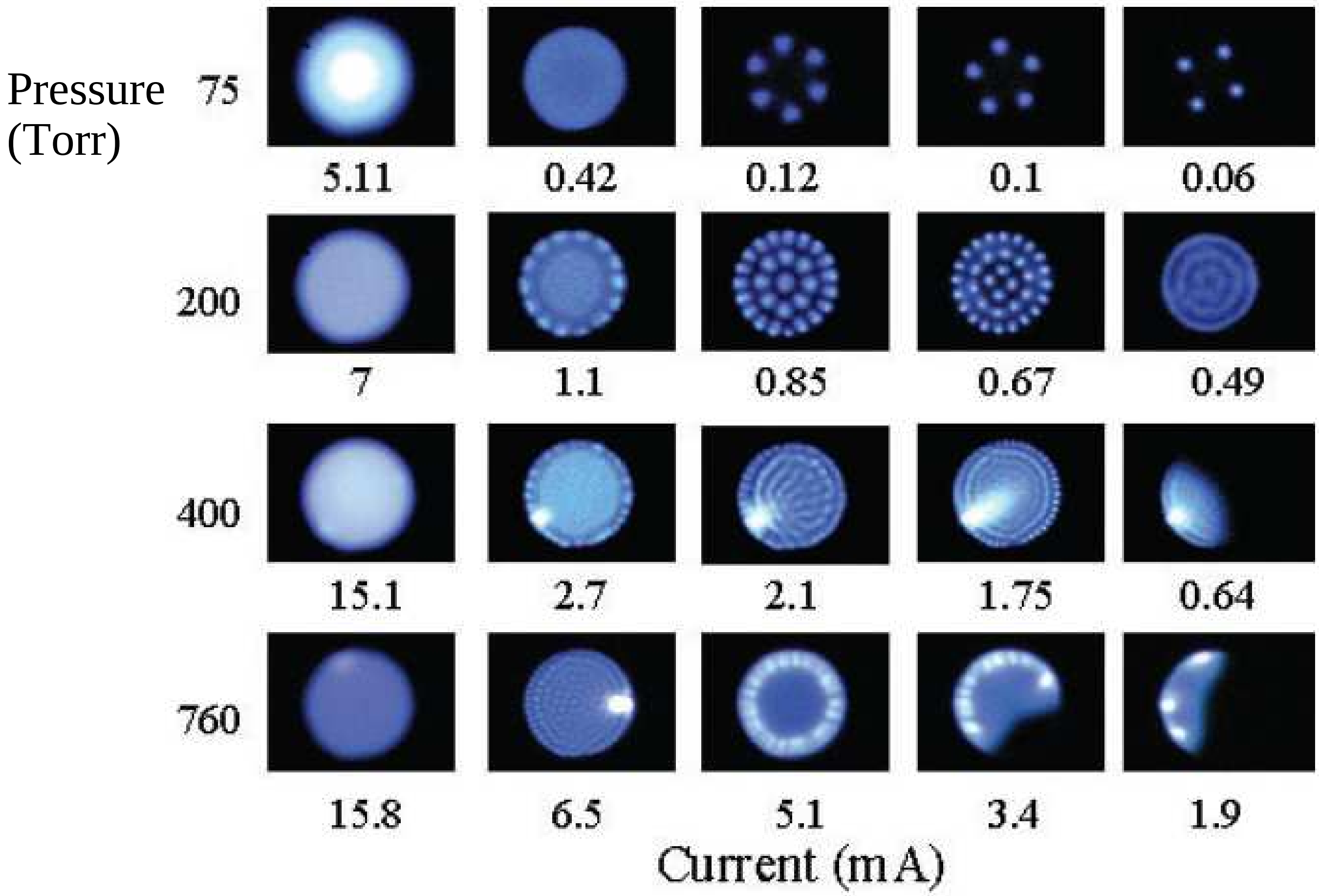
Spots on cathodes in high-pressure DC glows



End-on observation of a cathode of a glow discharge in xenon. R = 0.375 mm, d = 0.25 mm. From K. H. Schoenbach, M. Moselhy, and W. Shi 2004.

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Motivation for research

- **Arc plasma devices**: ensuring a desired mode of current transfer to cathodes is crucial and represents a major problem for designers. The diffuse mode is usually desirable.

  Example: arc (HID) lamps are critically affected by the evaporation of the electrode material, wall blackening, and electrode deformation.

- **Glow discharge devices**: spot modes may be desirable.

  Example: in APGD, $j_{min} \sim 10^5$ A/m^2 $\Longrightarrow$ spot mode is frequently needed.

  Example: the spots on cathodes of high-pressure glow microdischarges in Xe are sources of strong excimer emission.

A theoretical description of constricted current transfer to cathodes of DC discharges used to be one of the oldest unsolved problems of gas discharge physics.

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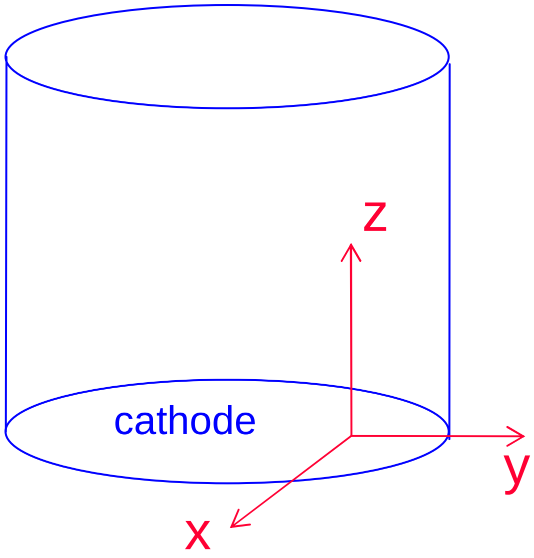
The approach

- An adequate theoretical model of current transfer to a DC cathode must not necessarily involve essentially different physical mechanisms.
- Rather, it must admit in some cases **multiple steady-state solutions** for the same discharge current, different solutions describing different modes of current transfer.



- **1D solution, *f=f(z)***

  The distributed mode [the diffuse mode on an arc cathode; the abnormal mode and the (unstable) mode corresponding to the falling branch of the CVC on the glow cathode].

- **Multidimensional solutions, *f=f(x,y,z)***
  Modes with spots.

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The approach

- The problem amounts to finding multiple solutions.
- Once this has been realized, is all the rest is a matter of numerical simulations?
- **If iterations in a nonlinear multidimensional problem with multiple solutions converge painlessly, then the converged solution is not the one that you need**!
- Enough qualitative information must be available in advance in order for multidimensional solutions describing spot modes to be computed. (It will also clarify the physics relevant and facilitate analysis of computation results.)
- This can be achieved by invoking the theory of self-organization in nonlinear dissipative systems.

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Contents of the talk

- Introduction
- **Modes of DC current transfer as predicted by general trends of self-organization in nonlinear dissipative systems**
- Particular models of current transfer: current transfer to arc cathodes
- Particular models of current transfer: current transfer to glow cathodes

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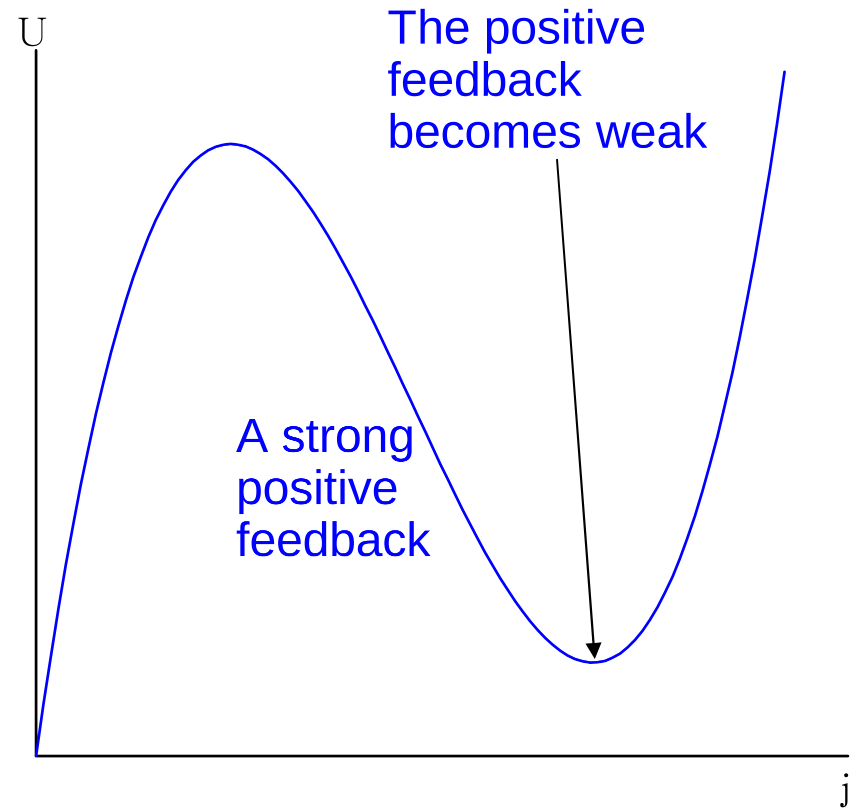
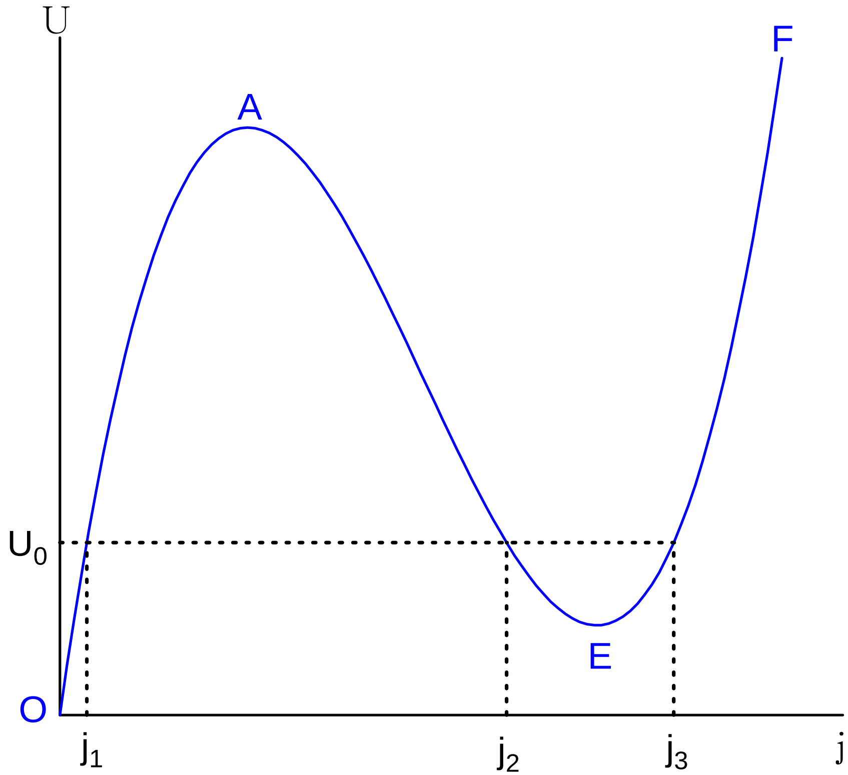
General predictions of the self-organization theory

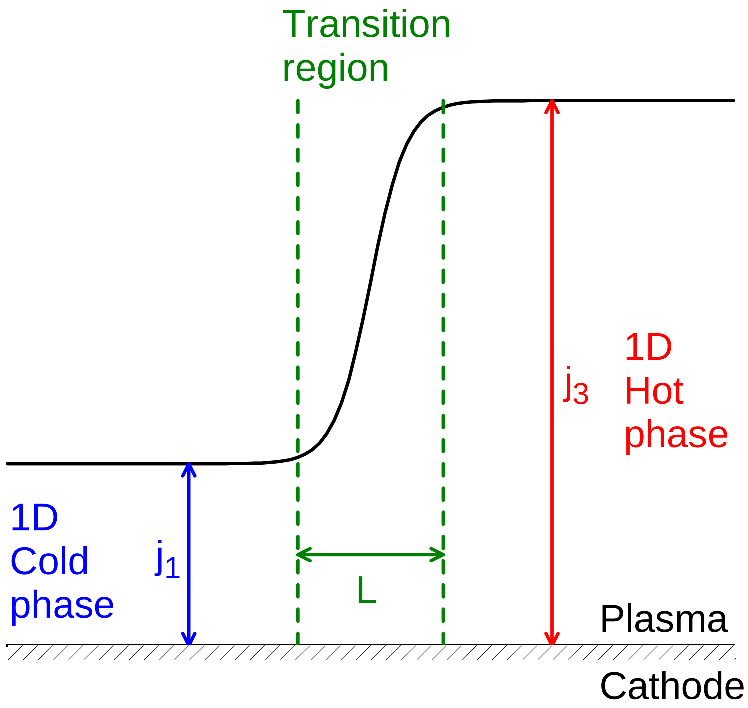
- Self-organization usually occurs in bi-stable systems, i.e., in those with an N-shaped characteristic.
- In our case, the characteristic that should be N-shaped is the CVC of the cathodic part of the discharge, *U(j)*, when the mode is distributed.



CVC of the distributed mode.

General predictions of the self-organization theory





***U= U₀*:** **Three 1D solutions**

$j=j_1$: stable (a cold phase),

$j=j_3$: stable (a hot phase),

$j=j_2$: unstable.

Multidimensional solutions: states with co-existence of phases, exist at a certain value of U_0 (Maxwell's construction) provided that cathode transversal dimensions >> L.

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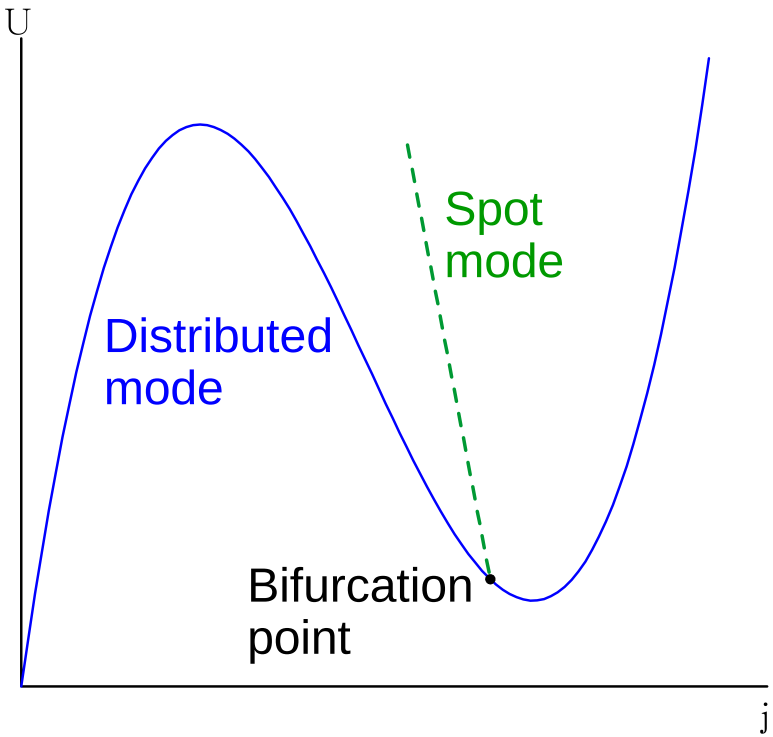
General predictions of the self-organization theory

- If multidimensional solutions exist, they branch off from the falling section of the current-voltage characteristic of the distributed mode.
- Points at which this branching occurs (bifurcation points) are points of neutral stability of the distributed mode.
- Bifurcation points may be found by means of linear stability analysis.



The procedure

1. To formulate a model of plasma-cathode interaction for the particular discharge. While being multidimensional in nature, this model must allow 1D solutions;

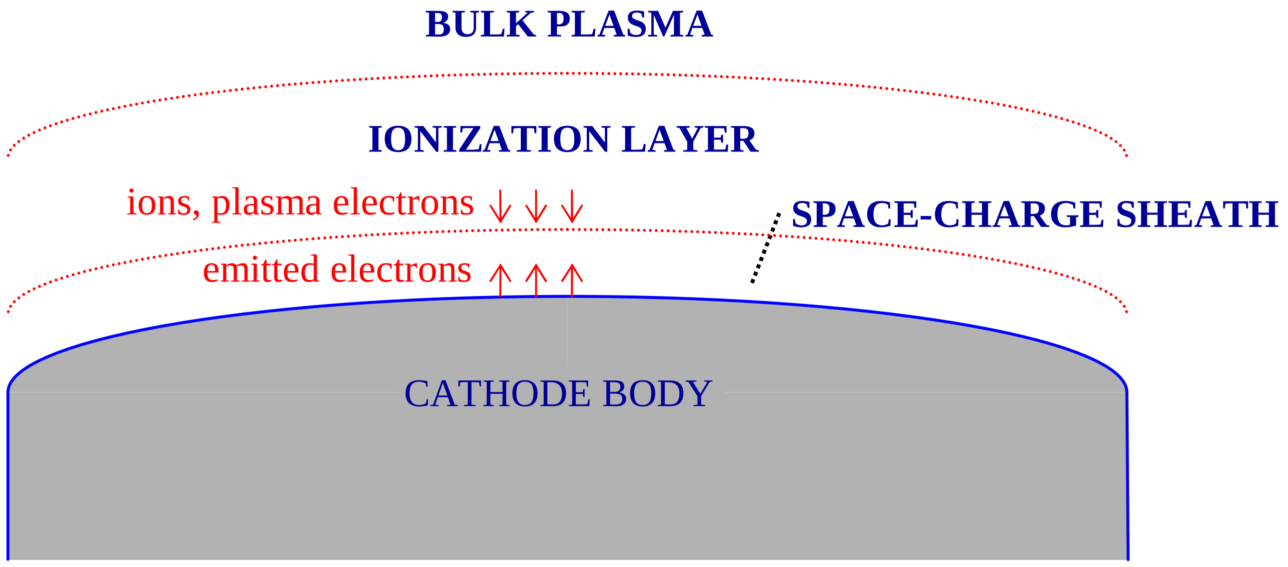
2. To find the 1D solution and the bifurcation points;

3. To find spot modes by means of numerical modeling with the use of results of the bifurcation analysis. To extend the theory in order to take into account the fact that the distributed mode is not precisely uniform;

4. To investigate stability of different modes.

Arc cathodes, step 1: the model

Structure of the near-cathode region in high-pressure arcs



	$T_e = T_h$	Ionization equilibrium	$n_e = n_i$	Description
Bulk plasma	No	Yes	Yes	-
Ionization layer	No	No	Yes	Fluid
Space-charge sheath	No	No	No	Kinetic

Arc cathodes, step 1: the model

The model of non-linear surface heating

A complete solution can be found in two steps:

- **Solution on the plasma side**: the 1D problem describing the current transfer across the near-cathode plasma layer is solved and all parameters of the layer are determined as functions of T_w and U. In particular, functions $q = q(T_w,U)$ and $j = j(T_w,U)$ are found.

- **Solution inside the cathode**: the equation of thermal conduction is solved with the boundary condition $q = q(T_w,U)$.

Arc cathodes, step 1: the model

The model of non-linear surface heating

- W. L. Bade and J. M. Yos 1963 (Technical Report of Avco Corporation);
- M. S. Benilov 1998;
- J. Wendelstorf 1999;
- S. Coulombe 2000;
- R. Bötticher and W. Bötticher 2000, 2001a, 2001b;
- T. Krücken 2001;
- W. Graser 2001;
- M. S. Benilov and M. D. Cunha 2002, 2003a, 2003b;
- R. Bötticher, W. Graser, and A. Kloss 2004;
- M. Galvez 2004;
- K. C. Paul *et al* 2004;
- G. M. J. F. Luijks, S. Nijdam, and H. A. v. Esveld 2005;
- L. Dabringhausen *et al* 2005;
- M. S. Benilov, M. Carpaij, and M. D. Cunha 2006;
- R. Bötticher and M. Kettlitz 2006;
- Advances in the experiment: J. Mentel *et al* (Bochum), M. Kettlitz *et al* (Greifswald).

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Arc cathodes, step 1: the model

Solution on the plasma side

The ionization layer: a hydrodynamics description

Equations

- Equation of motion of the ion fluid: ion inertia, ion pressure gradient, electric field force, friction forces due to elastic collisions of ions with neutral particles and to ionization of neutral particles.
- Equation of balance of the electron energy in the ionization layer.

Boundary conditions

- On the plasma side of the ionization layer: ionization equilibrium.
- On the "edge" of the space-charge sheath: the Bohm criterion.

The space-charge sheath: a kinetic description

Equations

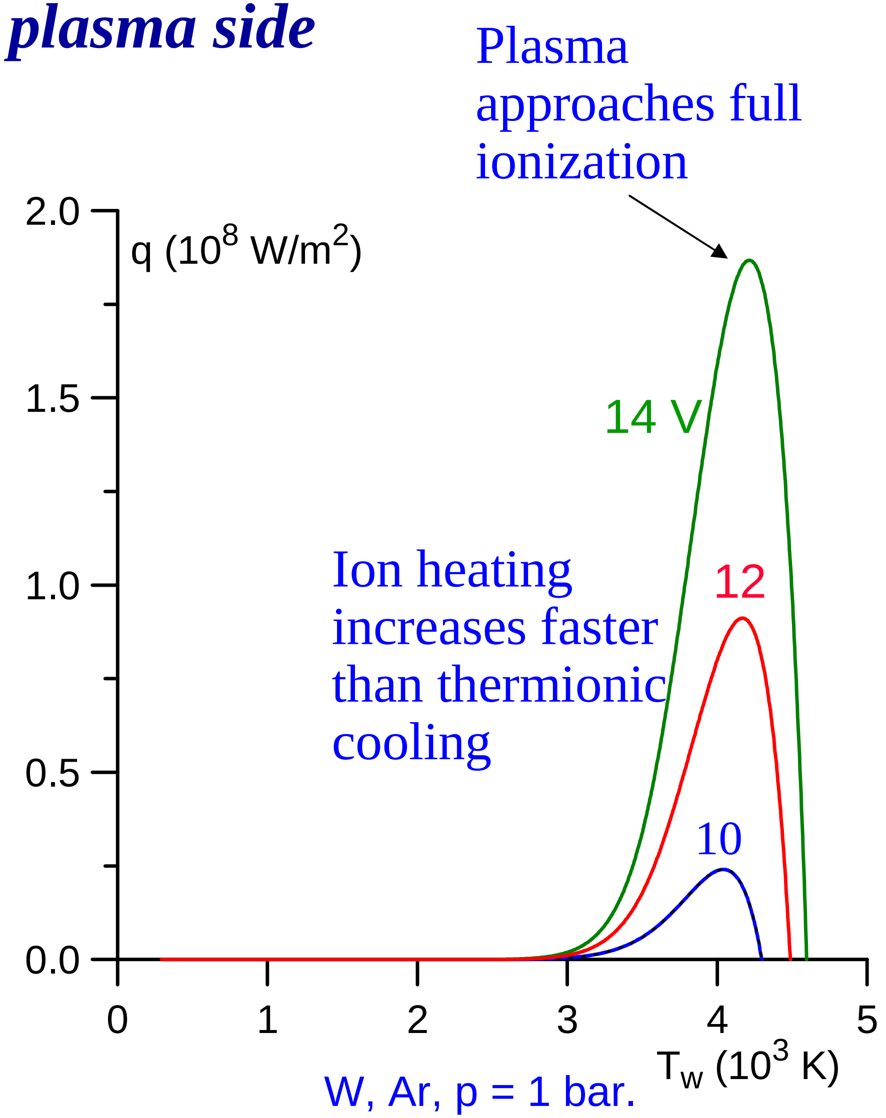
- A kinetic equation describing the motion of ions,
- The Poisson equation.

Boundary condition at the "sheath edge": the Bohm criterion

Arc cathodes, step 1: the model

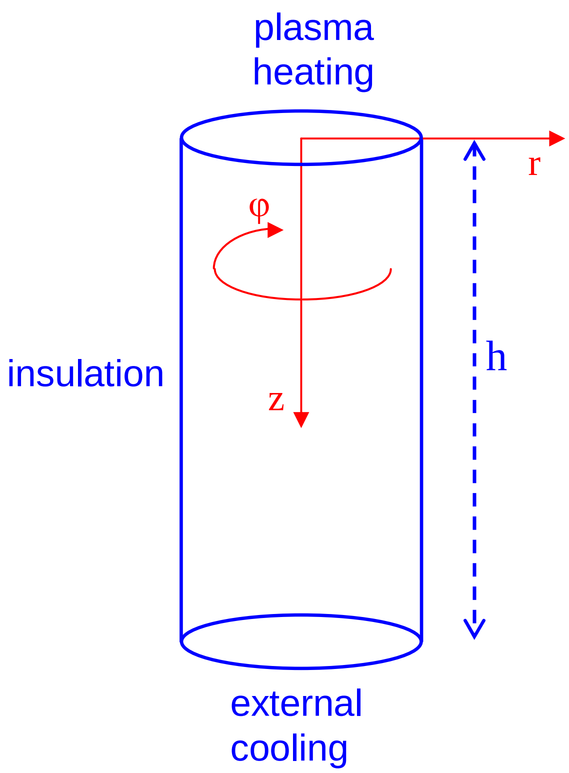
Solution on the plasma side

- The dependence of q on T_w is non-monotonic.
- In the conventional heat exchange, $q(T_w)$ is falling. **The rising section is a manifestation of a strong positive feedback.**

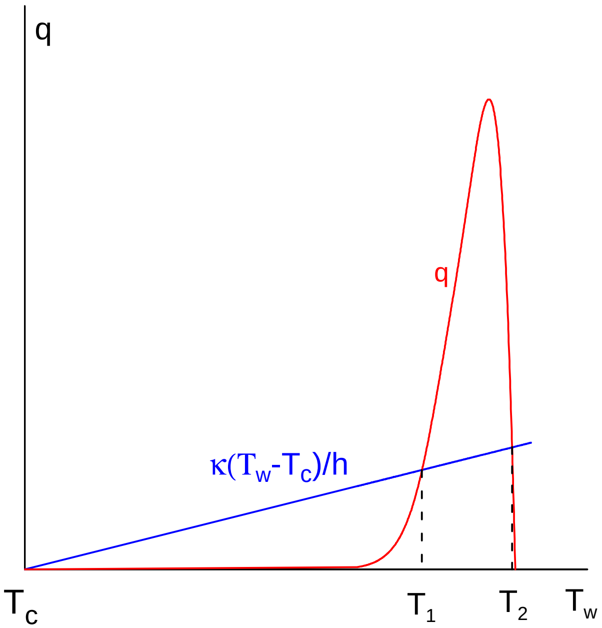


Arc cathodes, step 2: 1D solution

Cylindrical cathode with an insulated lateral surface



- **Spot modes**:
  2D solutions, $T = T(r,z)$,
  3D solutions, $T = T(r,z,\varphi)$.

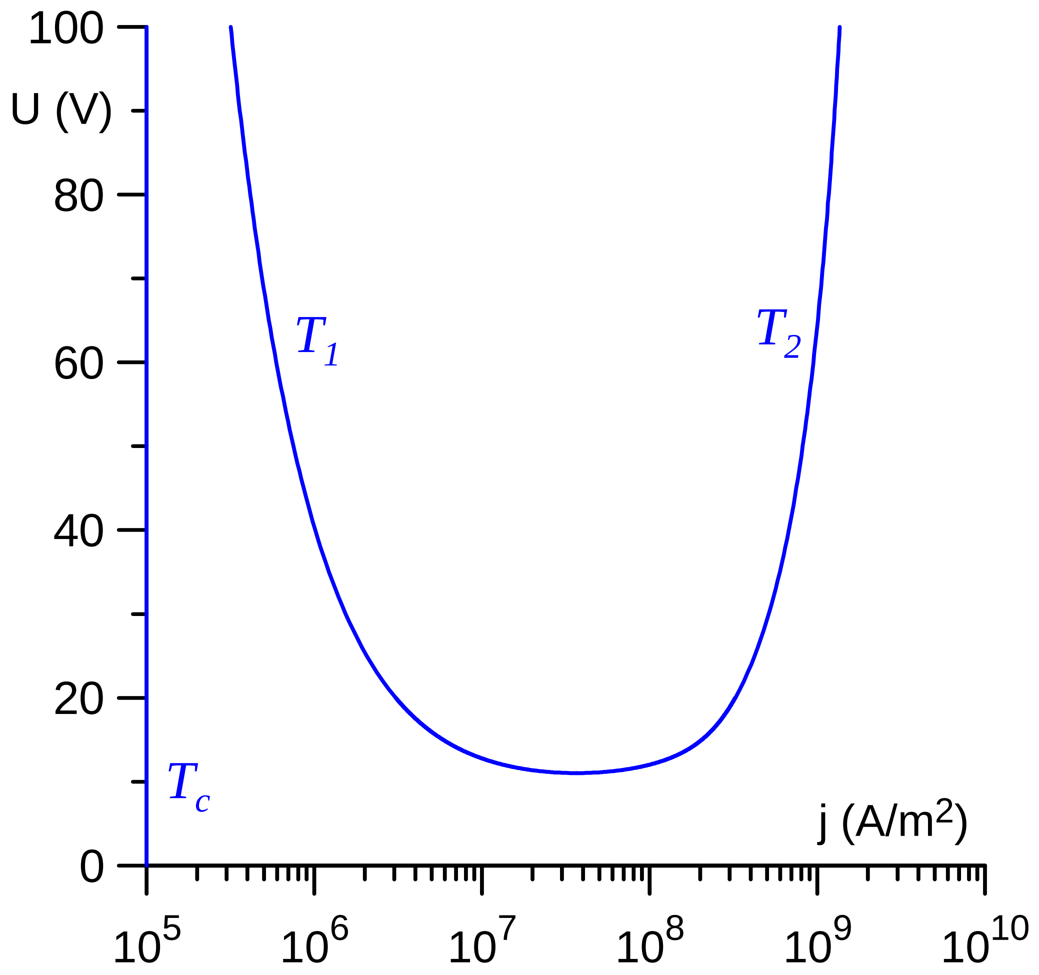


- **Diffuse mode**:
  1D solution, $T = T(z)$

1D thermal balance of the cathode.
Three roots: $T_c < T_1 < T_2$.

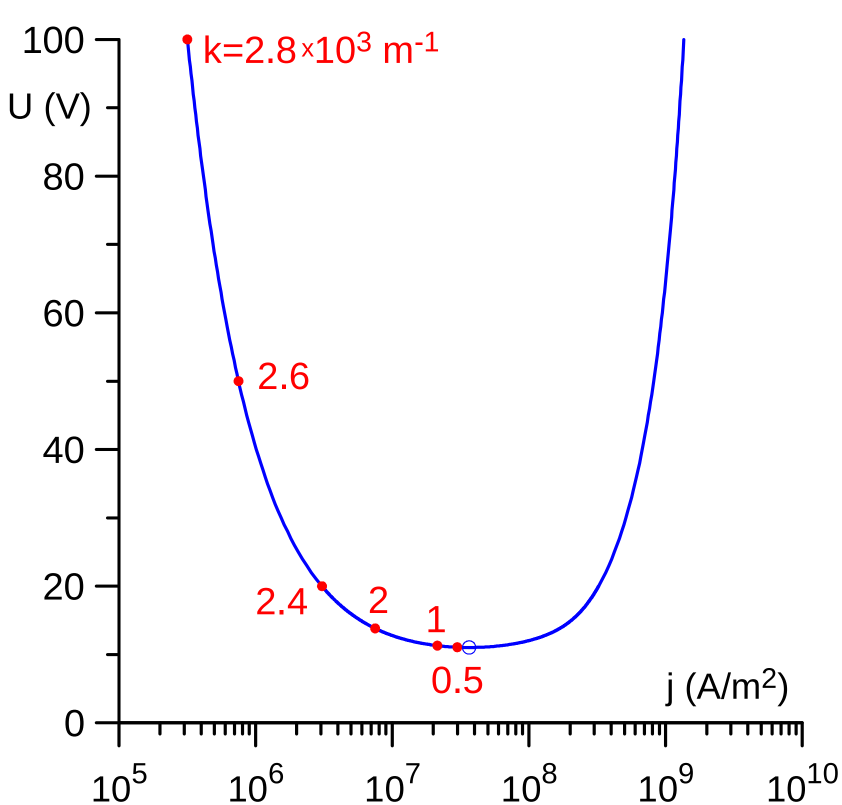
Arc cathodes, step 2: 1D solution

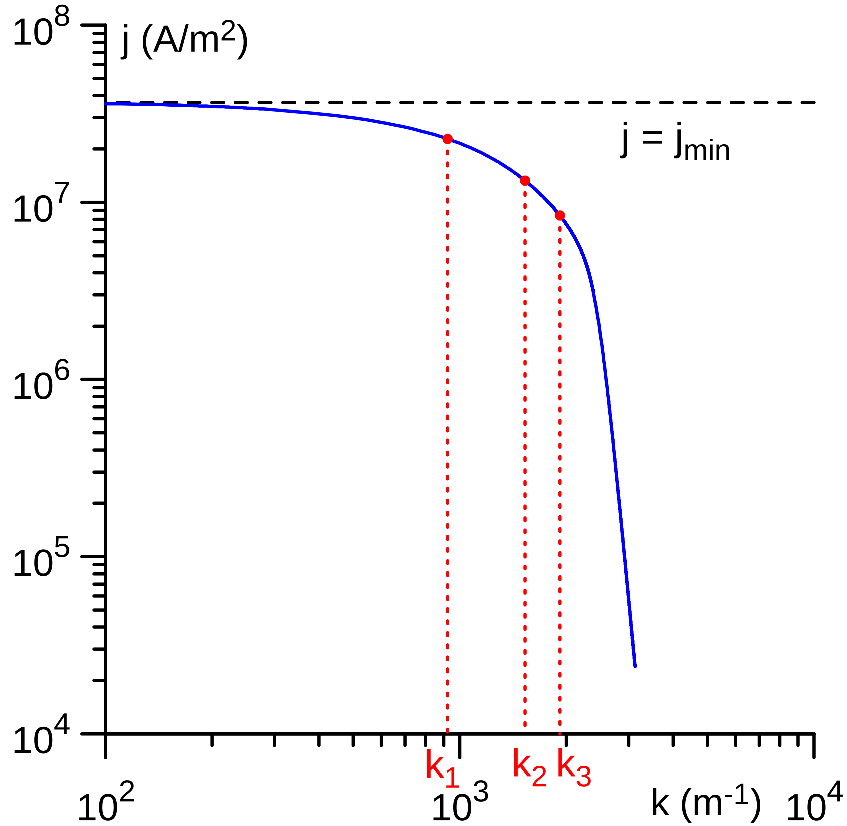
The CVC on the whole is N-shaped (rather than U-shaped).



CVC of the diffuse mode on a cylindrical cathode with an insulated lateral surface. W, h = 10 mm, Ar, 1 bar.

Arc cathodes, step 2: bifurcations





Circular cylinder

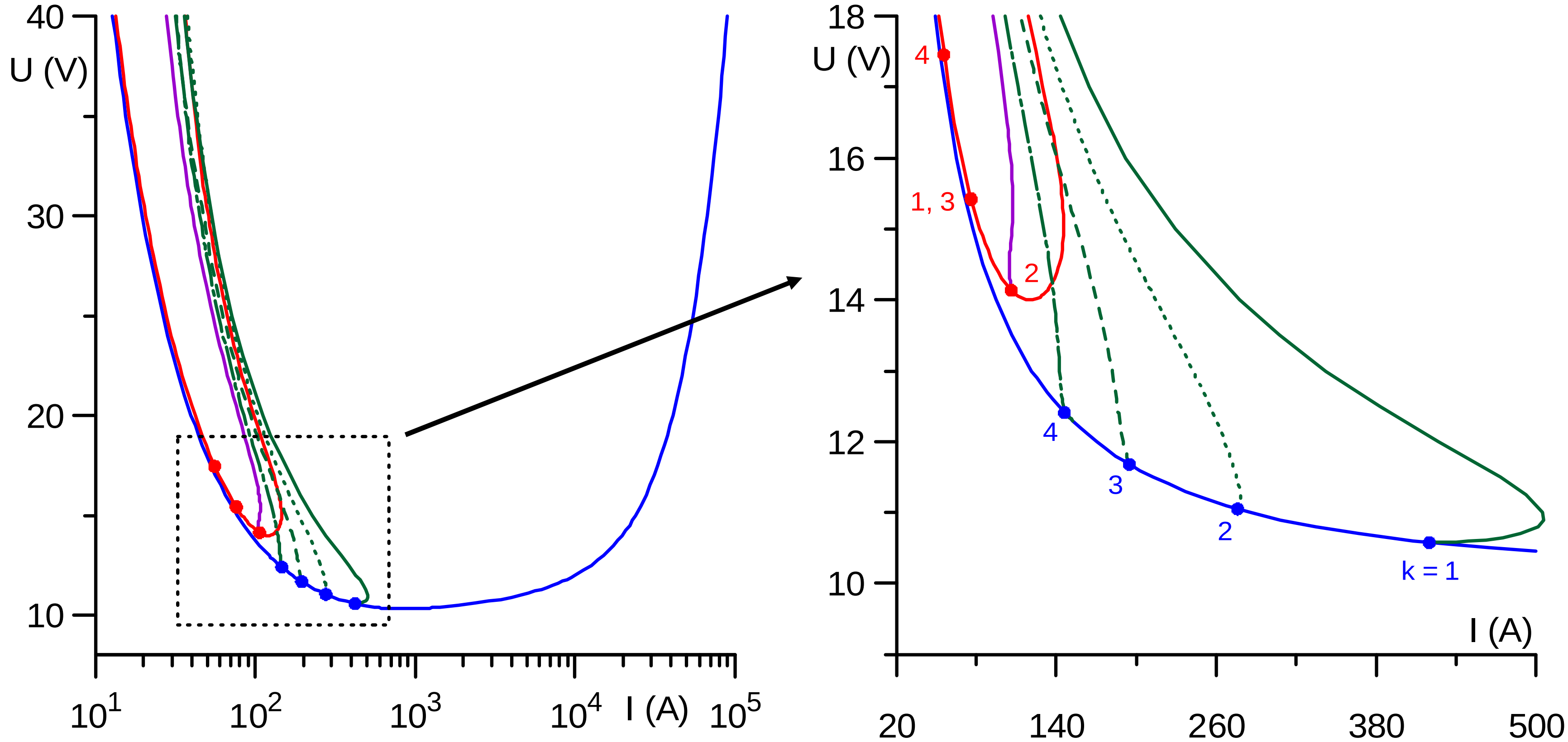
Diffuse mode on a cylindrical cathode with an insulated lateral surface. W, R = 2 mm, h = 10 mm, Ar, 1 bar.

$k_1 = 1.841/R$: 1D $\rightarrow$ 3D, a spot at the edge

$k_2 = 3.054/R$: 1D $\rightarrow$ 3D, two spots at the edge

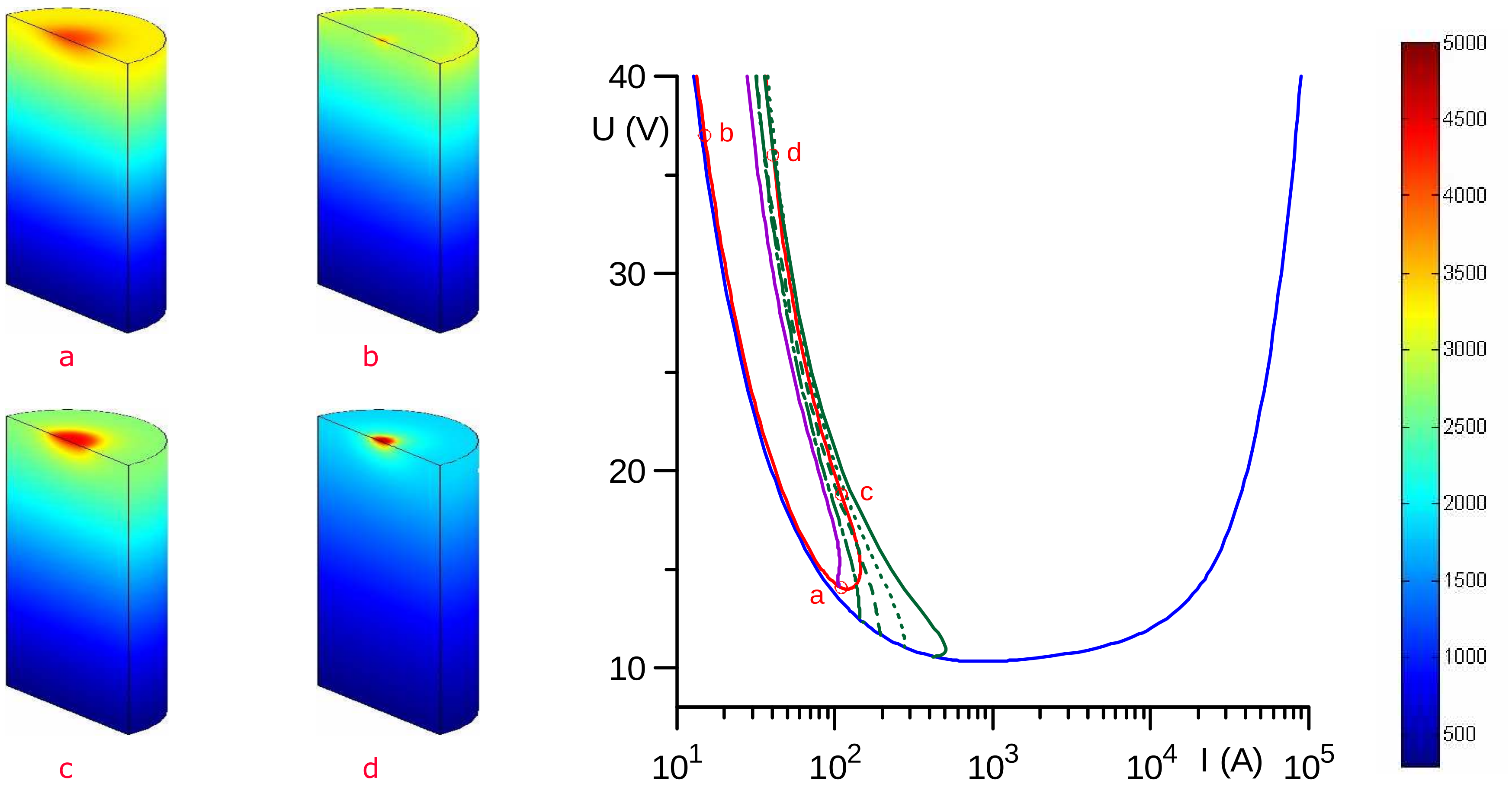
$k_3 = 3.832/R$: 1D $\rightarrow$ 2D, a spot at the center

Arc cathodes, step 3: general pattern of modes



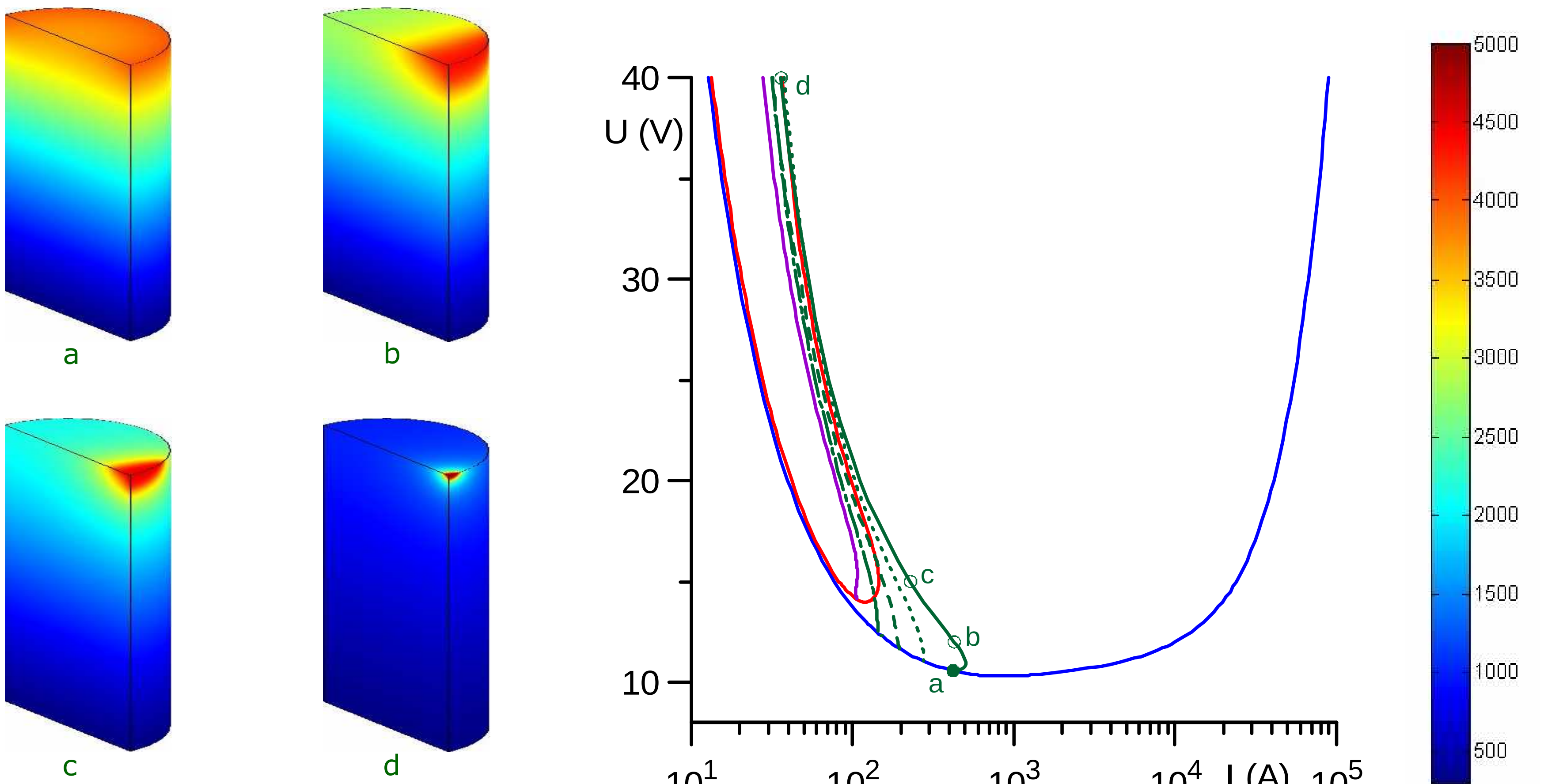
Current-voltage characteristics of different modes of current transfer. W, R = 2 mm, h = 10 mm, Ar, 1 bar. Blue: diffuse mode. Red: 1st 2D spot mode. Green: 3D spot modes which branch off from the diffuse mode. Purple: 3D spot mode which branches off from the 2D spot mode. ●, ●: bifurcation points.

Arc cathodes, step 3: mode with a central spot



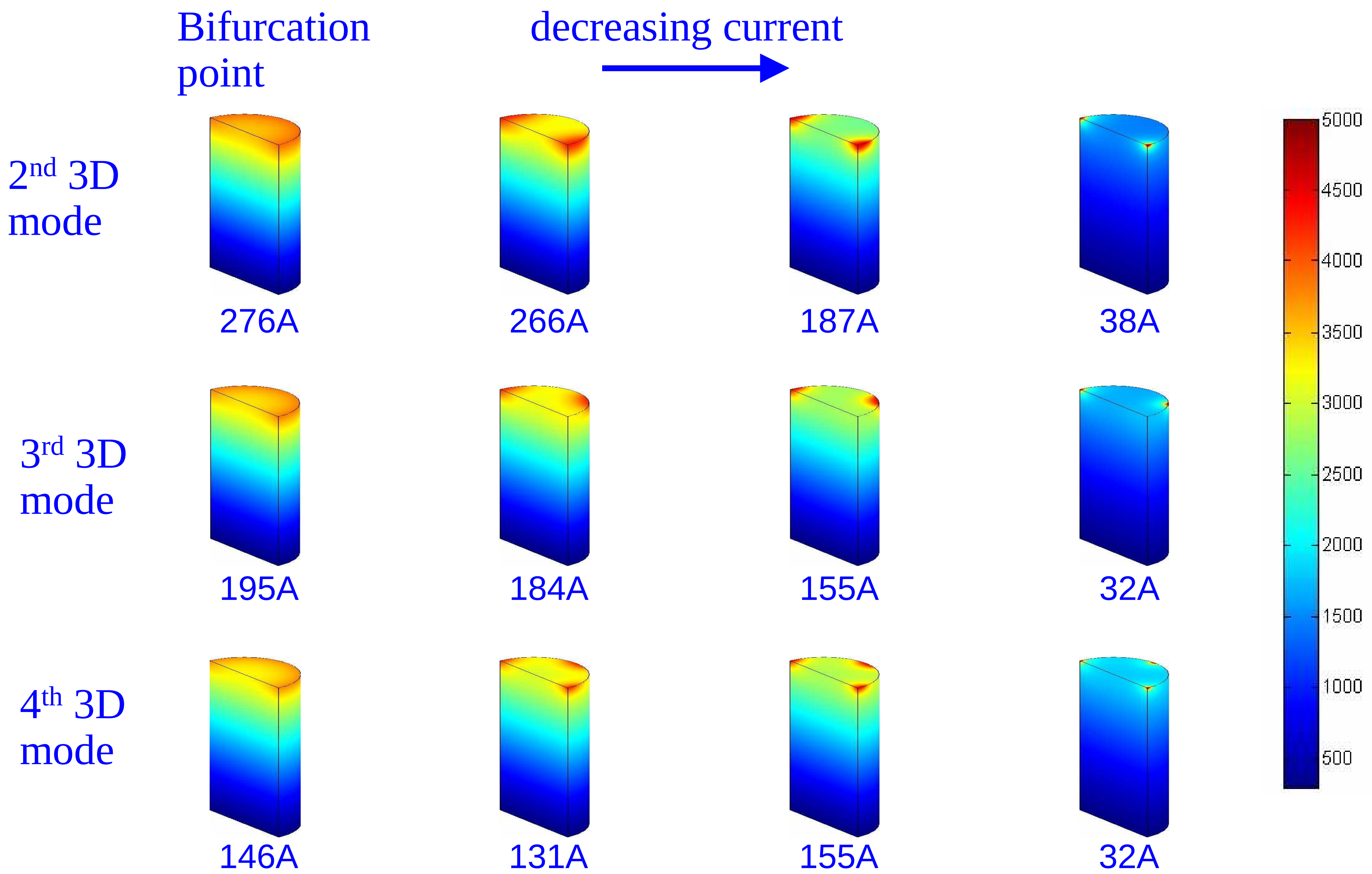
W, R = 2 mm, h = 10 mm, Ar, 1 bar. From M. S. Benilov, M. Carpaij, and M. D. Cunha 2006.

Arc cathodes, step 3: mode with one edge spot



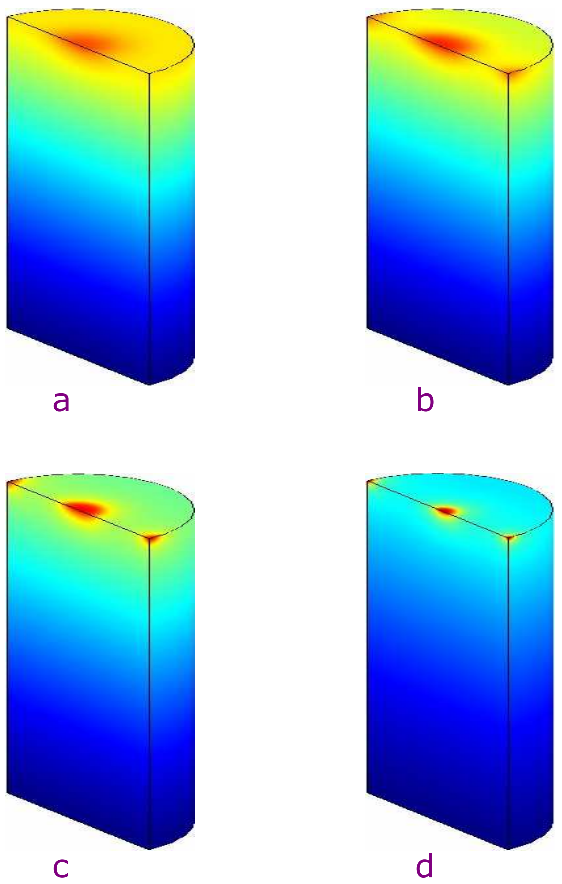
W, R = 2 mm, h = 10 mm, Ar, 1 bar. From M. S. Benilov, M. Carpaij, and M. D. Cunha 2006.

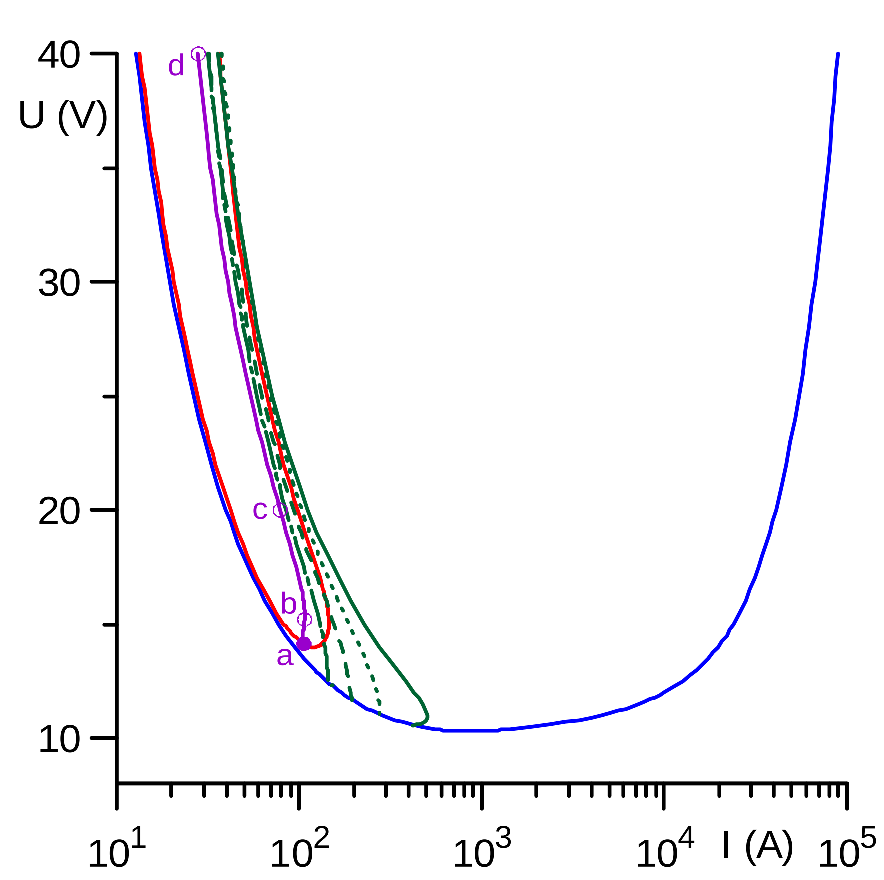
Arc cathodes, step 3: modes with multiple edge spots

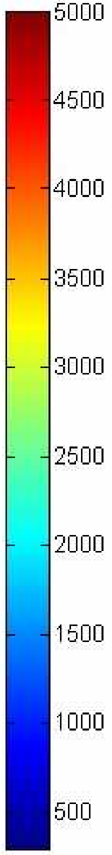


W, R = 2 mm, h = 10 mm, Ar 1 bar. From M. S. Benilov, M. Carpaij, and M. D. Cunha 2006.

Arc cathodes, step 3: central spot and two edge spots



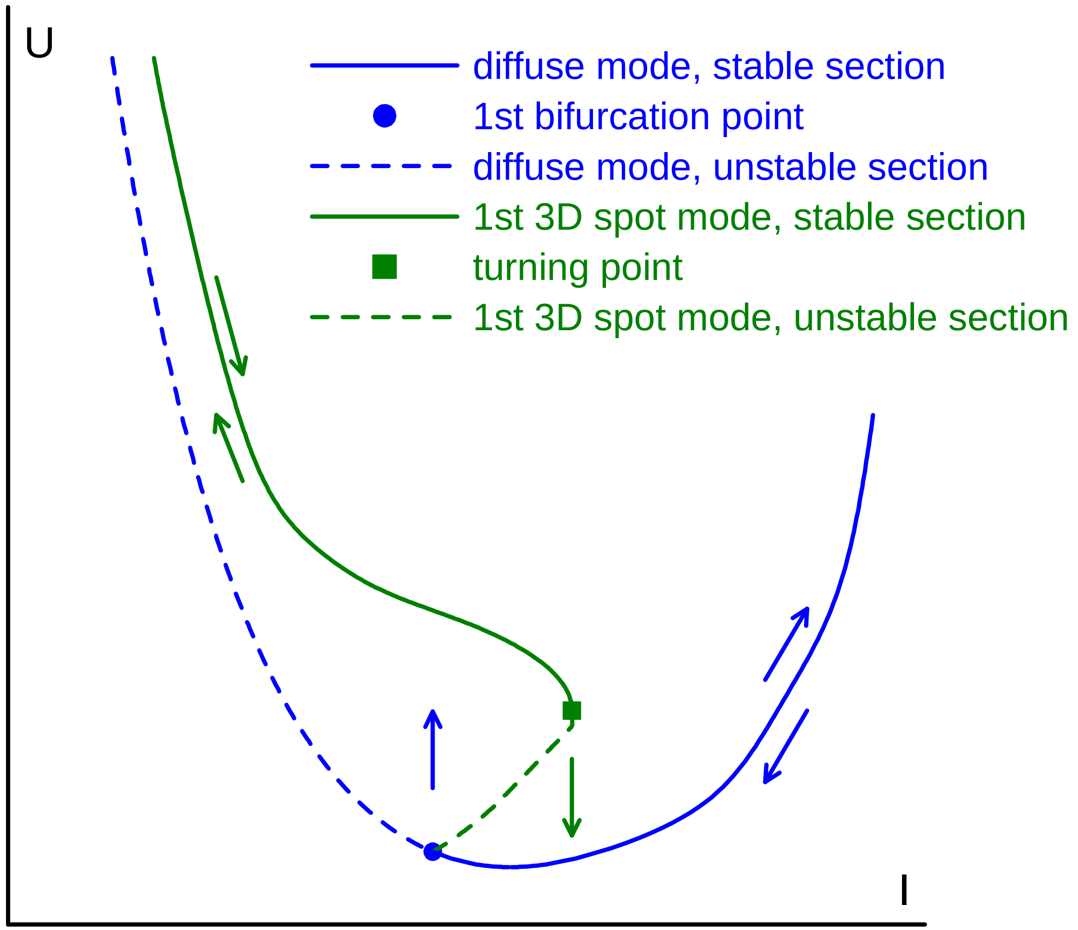




W, R = 2 mm, h = 10 mm, Ar, 1 bar. From M. S. Benilov, M. Carpaij, and M. D. Cunha 2006.

Arc cathodes, step 4: stability of different modes

Analytical and numerical treatment has confirmed assumptions based on general trends of self-organization in nonlinear dissipative systems:



The second and all the subsequent spot modes are unstable.

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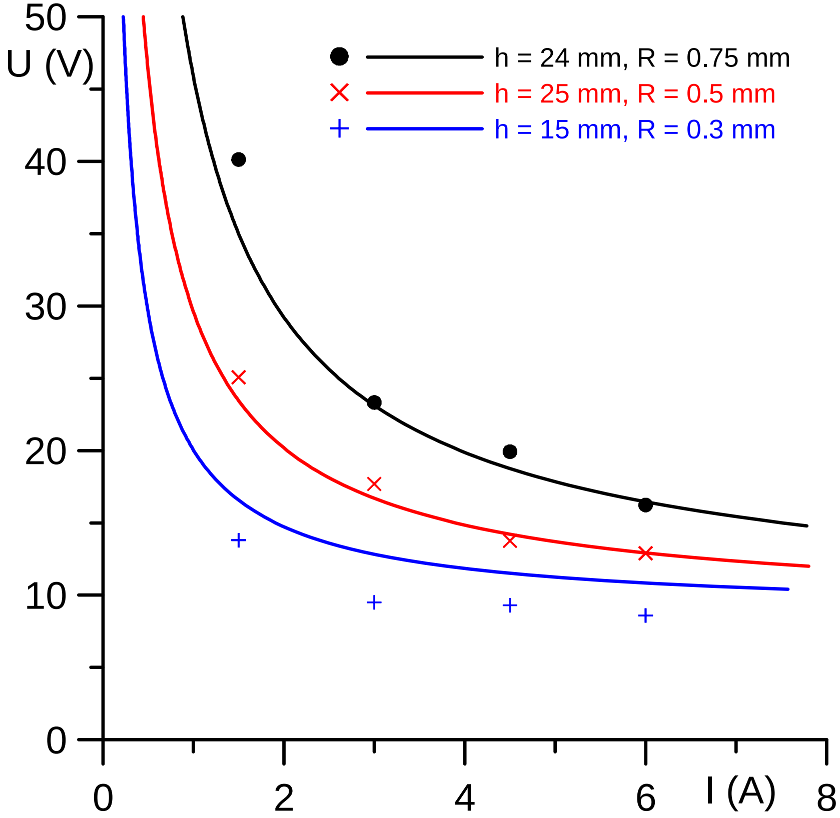
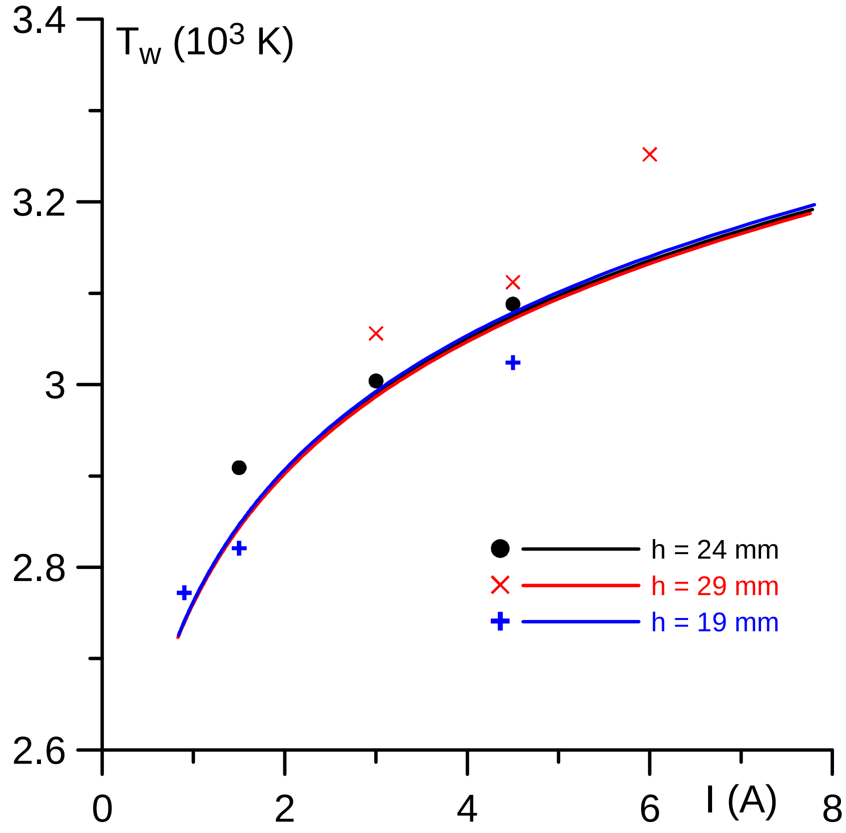
Comparison with the experiment, diffuse mode

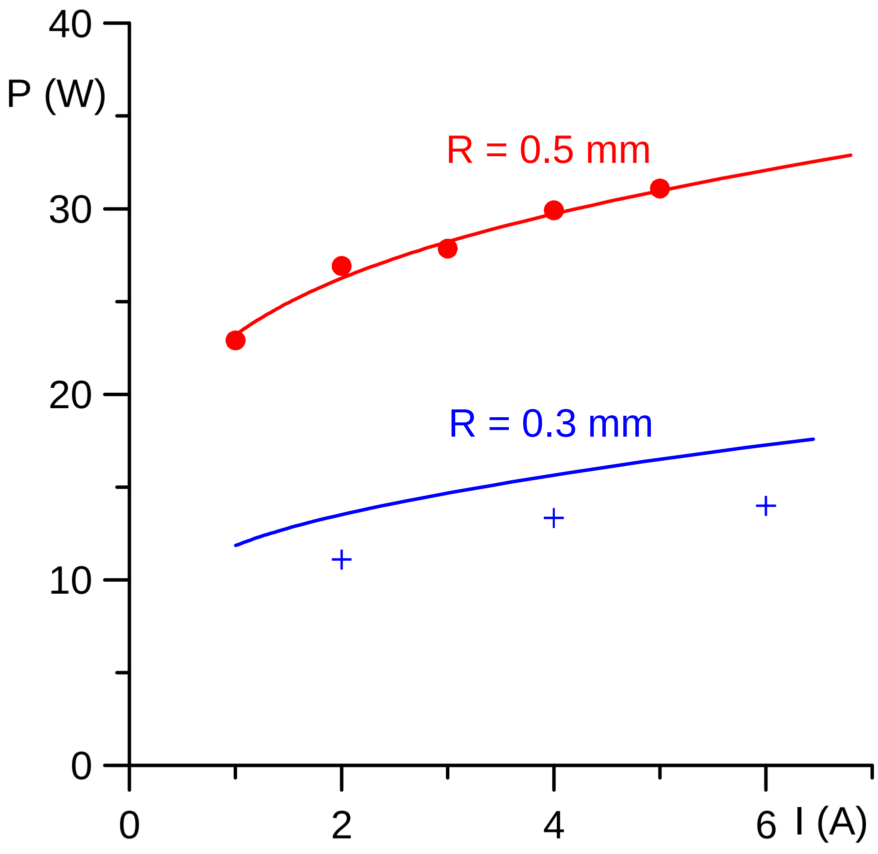




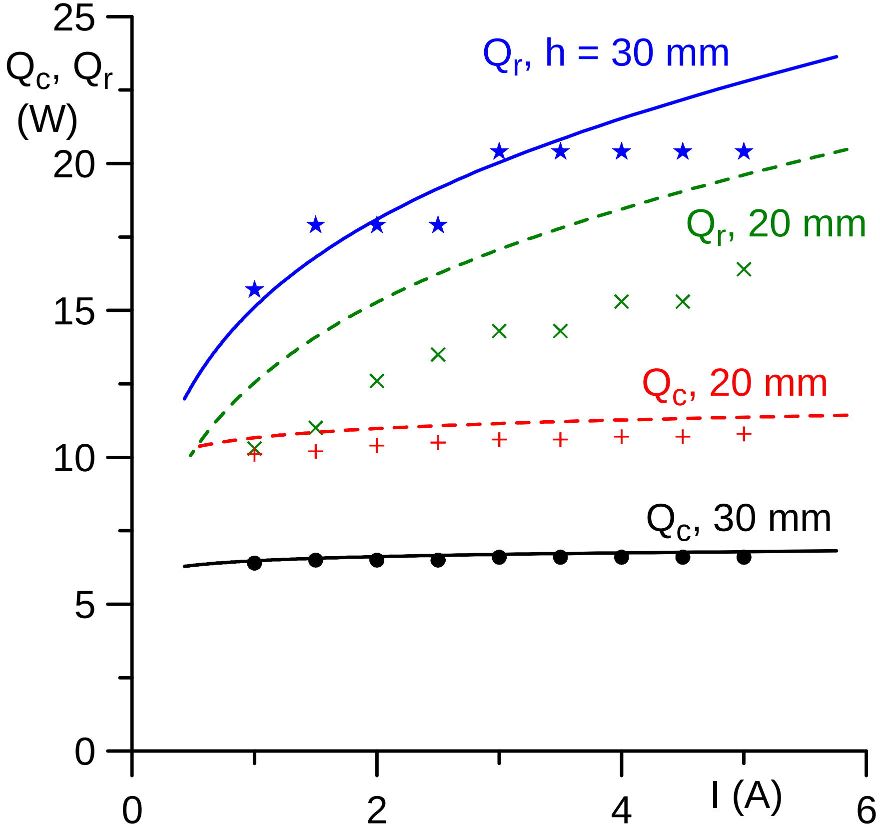
R = 0.75 mm

W, Ar, 2.6 bar. From M. S. Benilov and M. D. Cunha 2002.

Comparison with the experiment, diffuse mode



W, h = 20 mm, Ar, p = 2.6 atm.
From D. Nandelsätdt *et all* 2002.



W, R = 0.5 mm, Ar, p = 2.6 atm. From
M. S. Benilov and M. D. Cunha 2002.

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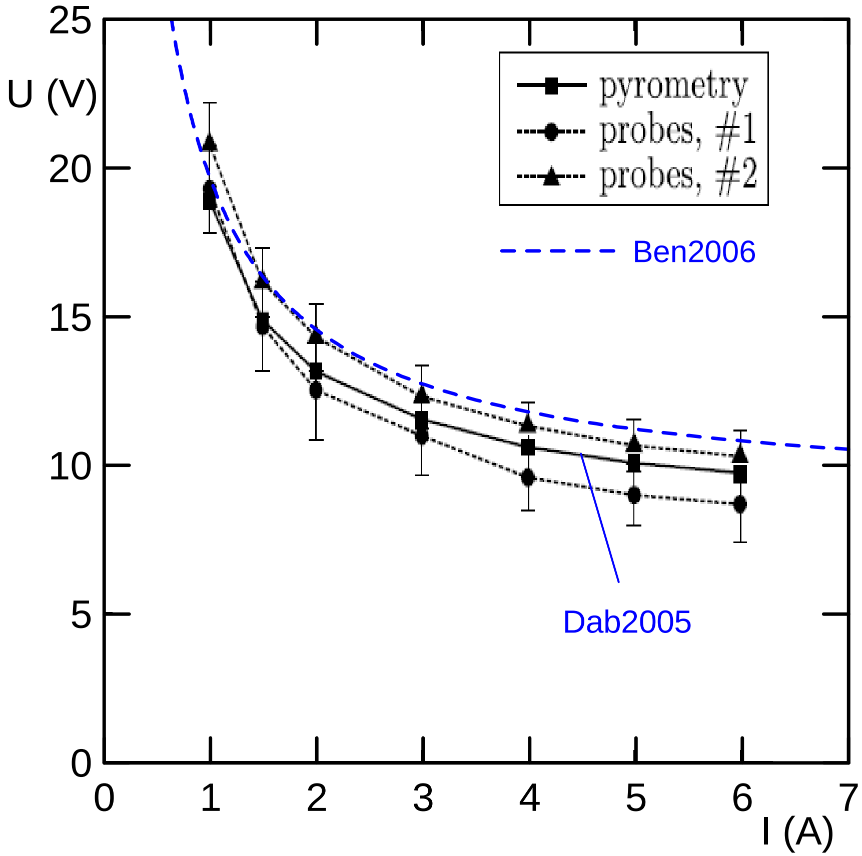
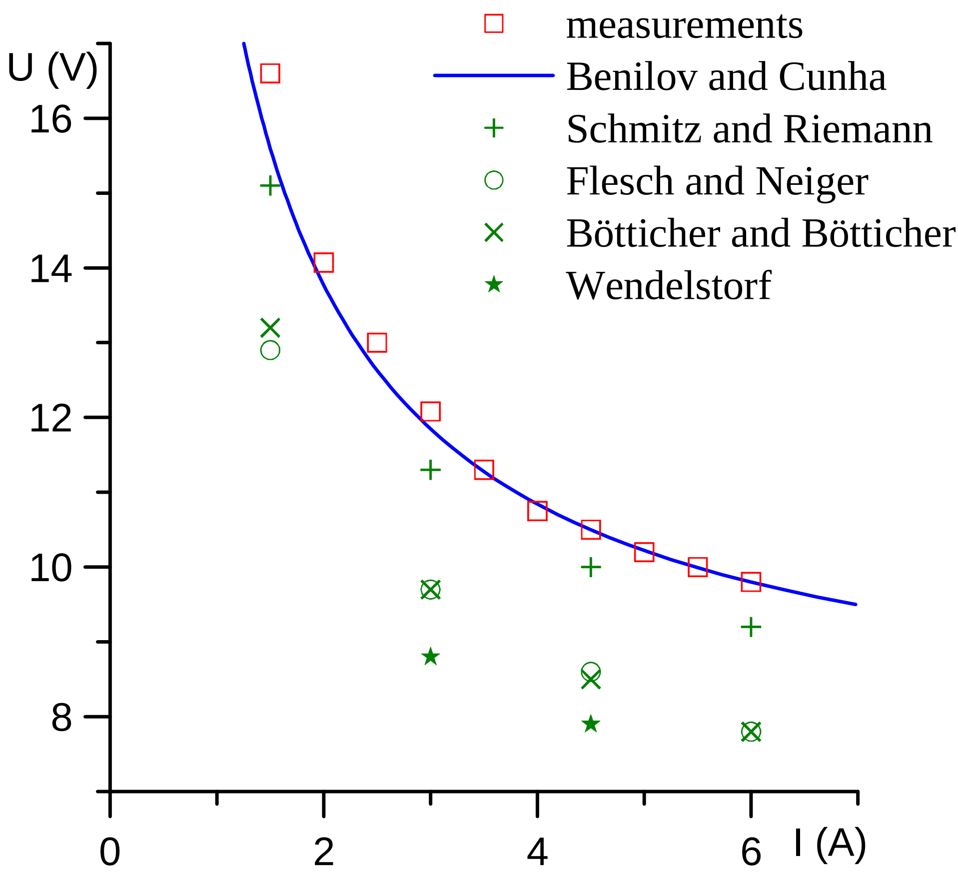
Comparison with the experiment, diffuse mode





W, R = 0.3 mm, h = 20 mm, Ar, p = 2.6 bar. From L. Dabringhausen *et al* 2005 and M. S. Benilov, M. Carpaij, and M. D. Cunha 2006.

W, R = 0.6 mm, h = 14 mm, Xe, p = 10 atm. From D. Nandelsätdt et all 2002.

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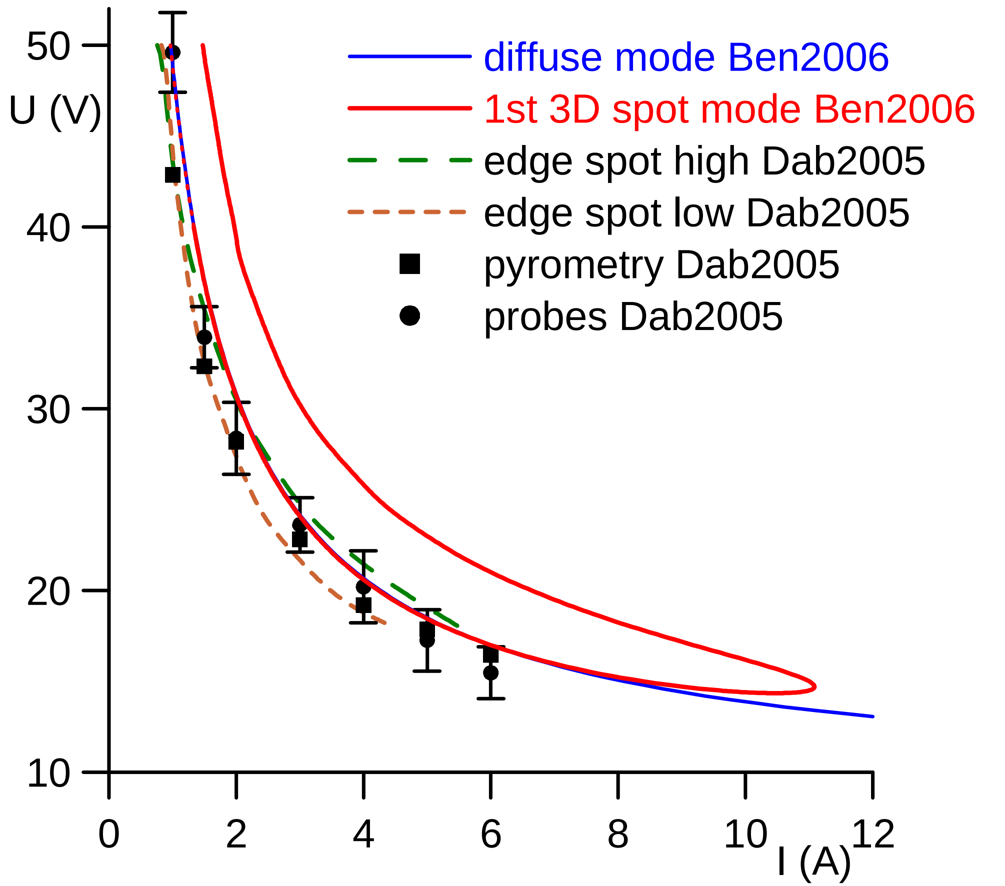
Comparison with the experiment, spot mode

- Both the diffuse mode and the high-voltage branch of the 1st spot mode are stable in the experimental current range.
- This is why the transition between diffuse and spot modes is so difficult to be reproduced in the experiment.



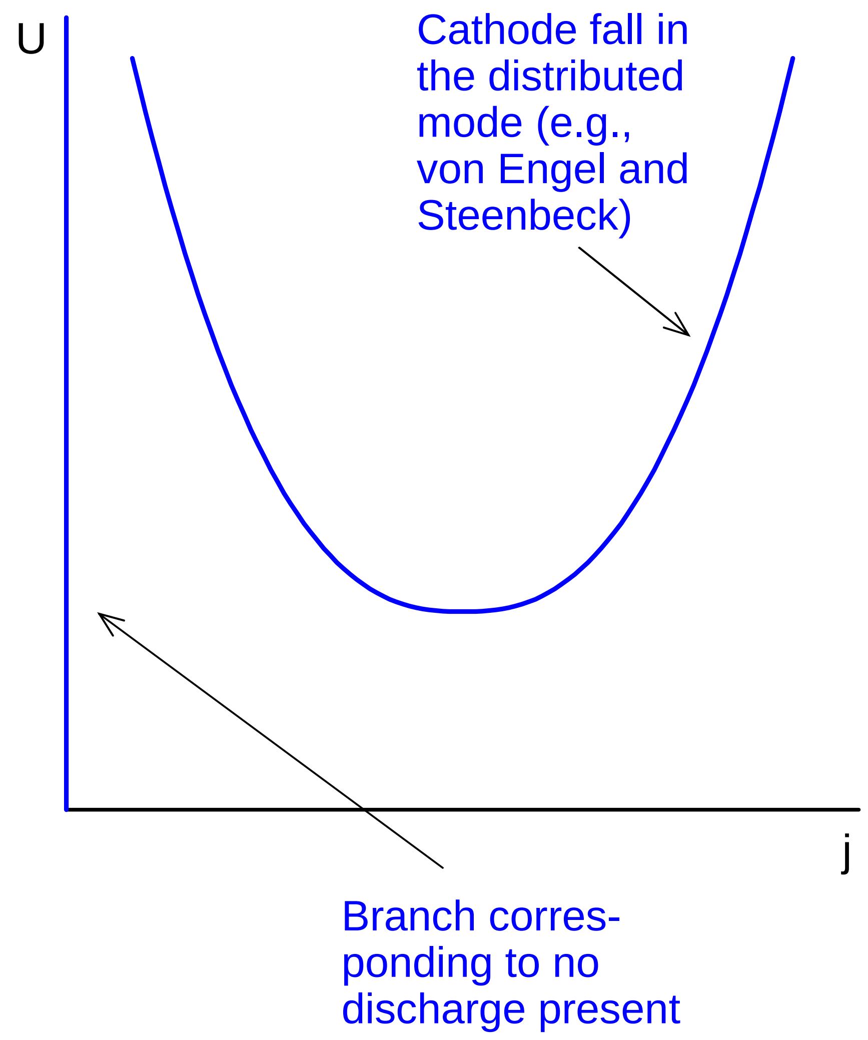
W, R = 0.75 mm, h = 20 mm, Ar, 2.6 bar.
From L. Dabringhausen *et al* 2005 and M. S. Benilov, M. Carpaij, and M. D. Cunha 2006.

There is a lot more to say …

- Modeling and experimental investigation of transient effects: very interesting dynamic mode changes;
- Modeling of cathodes of a complex shape made of different materials;
- Modeling of multispecies plasmas with complex chemical kinetics (air, plasmas of metal halides);
- Variation of the work function due to deposition of a monoatomic layer of an alkali metal: N-S-shaped CVC of the diffuse mode;
- Theory of solitary spots, i.e., spots on large cathodes: the spot radius was self-consistently determined by means of an appropriate Maxwell's construction.

Glow cathodes, step 1: the model

- The cathode fall of a DC glow is U-shaped.
- The falling section: the ionization coefficient rapidly increases ═► **a strong positive feedback.**
- The growing section: the ionization coefficient approaches saturation.
- The CVC of the near-cathode layer on the whole is N-shaped. **No additional mechanisms are required to describe spot modes.**



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Glow cathodes, step 1: the model

The system of differential equations

$$\nabla \cdot \left(-\mu_i n_i \nabla \varphi \right) = \dot{n}_e$$

$$\nabla \cdot \left(\mu_e n_e \nabla \varphi \right) = \dot{n}_e$$

$$\varepsilon_0 \nabla^2 \varphi = e\left(n_e - n_i \right)$$

$$\dot{n}_e = \nu_i n_e - \nu_{dif} n_e - \beta_{dis} n_i n_e$$

ν_i : the ionization frequency

ν_{dif}: the effective frequency of diffusion losses

β_{dis}: the coefficient of dissociative recombination

Glow cathodes, step 1: the model

The boundary conditions

On the cathode surface: $\varphi = 0, \quad \mu_e n_e = \gamma \mu_i n_i$

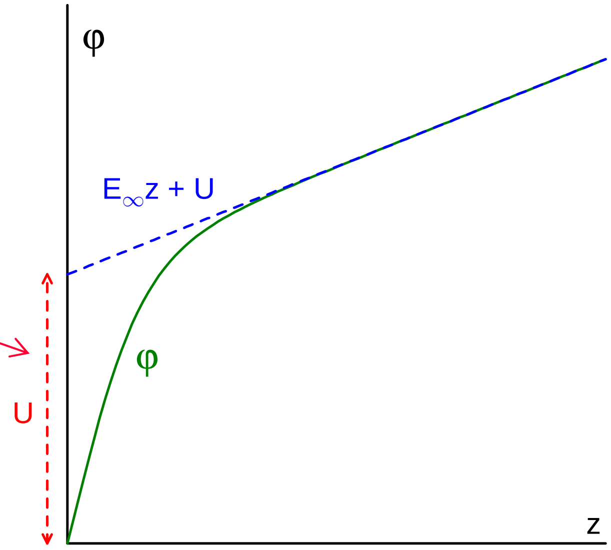
On the lateral surface of the tube: $\dfrac{\partial \varphi}{\partial n} = 0$

On the plasma side of the sheath: $E = E_\infty, \quad \varphi = E_\infty z + U$

E_∞ : the electric field in the plasma column

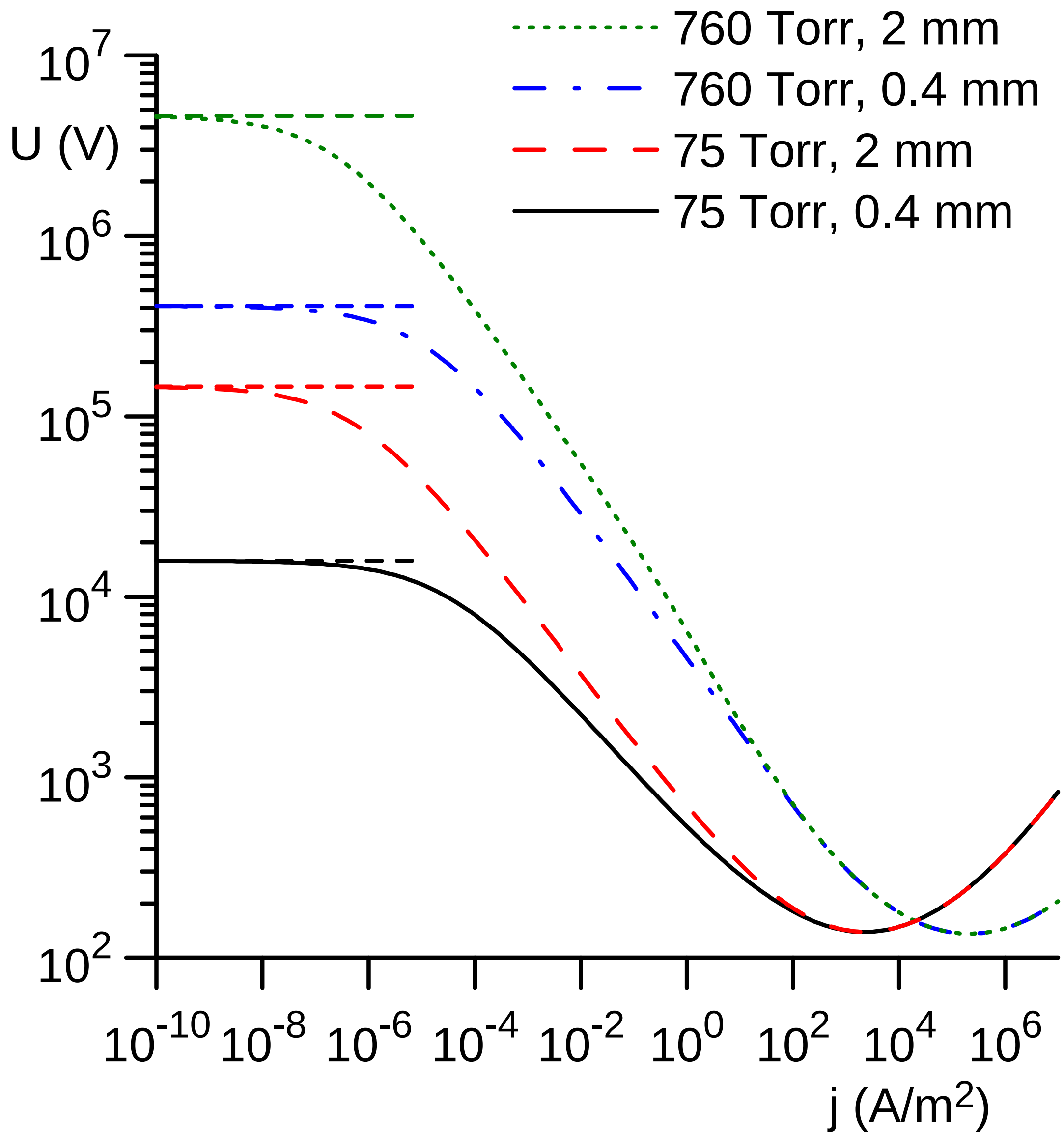
U : the sheath voltage

One of the two quantities E_∞ and U is given and the other is found as a part of the solution.



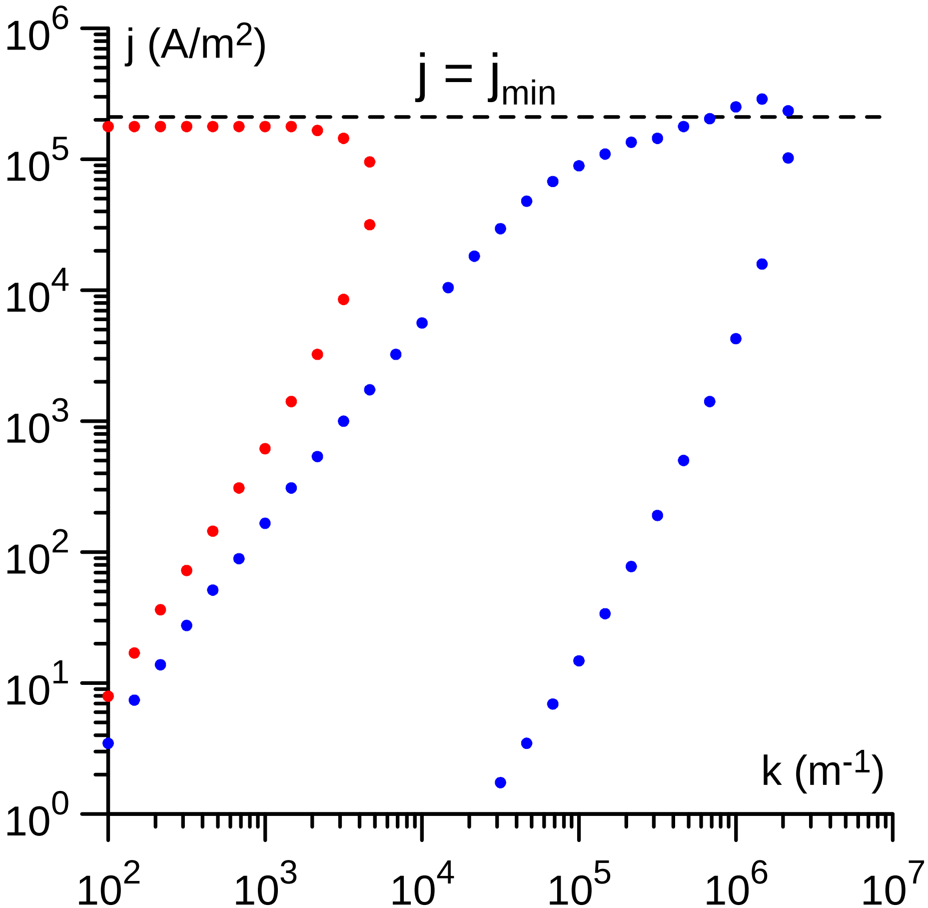
Matching plasma-sheath is smooth - no arbitrarily defined "sheath edge"

Glow cathodes, step 2: 1D solution



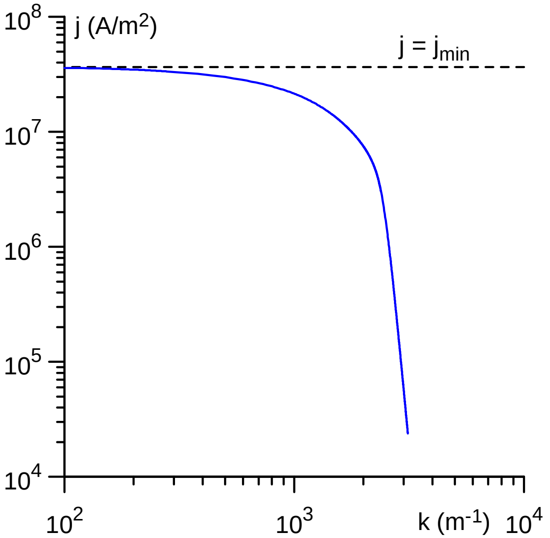
Current-voltage characteristics of the near-cathode sheath for two different plasma pressures and two different cathode radii. Xe, γ = 0.03.

Glow cathodes, step 2: Bifurcation points

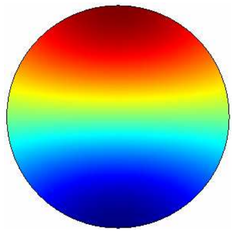


Xe, 760 Torr, γ = 0.03, *R* = 0.4 mm.

- As *j* decreases, patterns appear on the growing section of the CVC.
- **Appearing paterns comprise many spots.** Very different from patterns appearing on arc cathodes!



Glow cathodes, step 2: Patterns of spot modes



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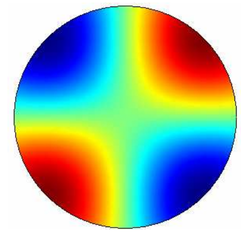

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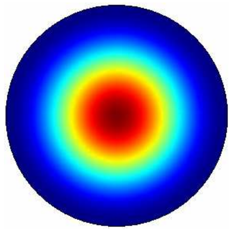
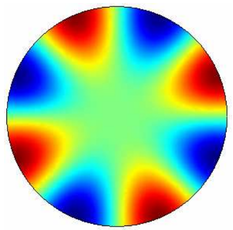


0,2



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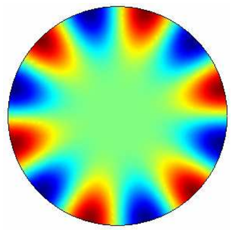
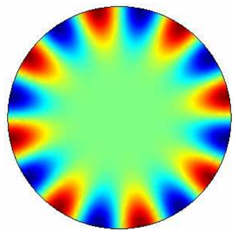
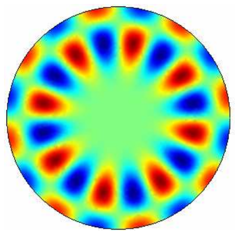


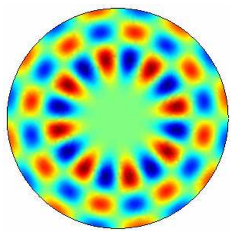
6,1



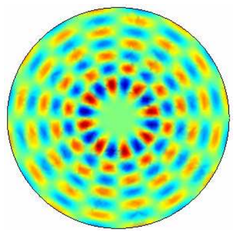
8,1



8,2



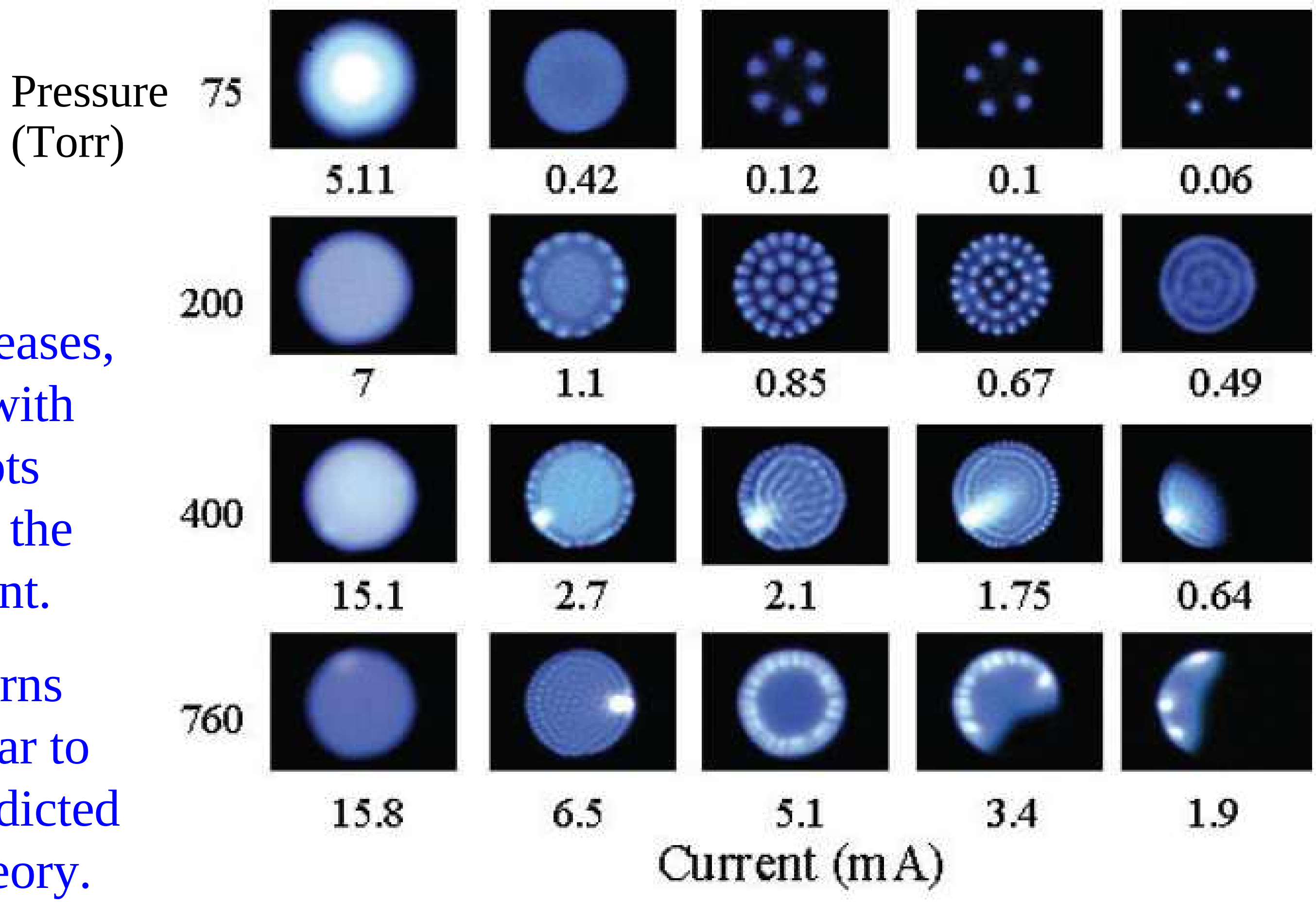
8,3



8,7

Patterns on cathode of high-pressure DC glow in Xe

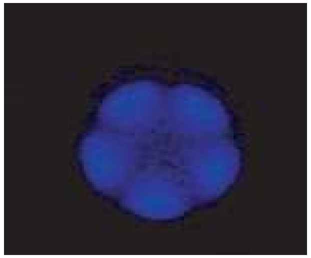
- As *j* decreases, patterns with many spots appear in the experiment.
- The patterns are sumilar to those predicted by the theory.



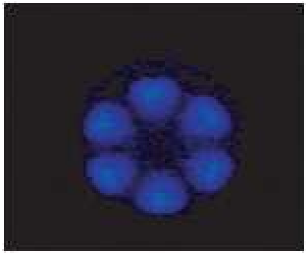
From K. H. Schoenbach, M. Moselhy, and W. Shi 2004.

Patterns on high-pressure glow and arc cathodes

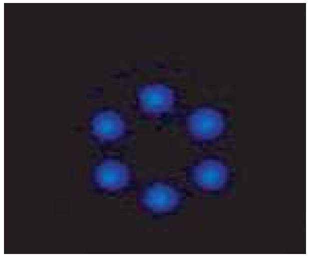
0.9 mA



0.17 mA

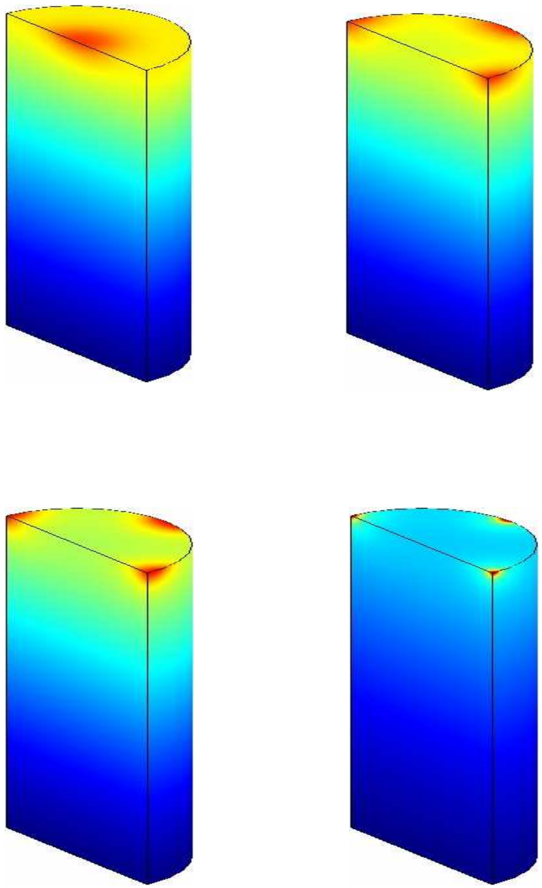


0.16 mA



0.11 mA

End-on observation of a W cathode of a glow discharge in Xe. p = 75 Torr, R = 0.375 mm, d = 0.25 mm. From N. Takano and K. H. Schoenbach 2006.



Simulation of a W cathode of an arc discharge in Ar. p = 1 bar, R = 2 mm, h = 10 mm. From M. S. Benilov, M. D. Cunha and M. Carpaij 2006.

Conclusions

Spots on high-pressure arc cathodes

- A complete theory has been developed and validated by the experiment. **http://www.arc_cathode.uma.pt:** a free on-line tool for simulation of diffuse mode on cathodes of high-pressure arc discharges.

Spots on DC glow cathodes

- A theory may be developed by along the same lines. Only initial steps on this way are being made.
- Given a number of features in common, work in this field will be favored by advances achieved in understanding of spot modes on arc cathodes.

Stationary patterns in other discharges?

- Spots on cathodes of vacuum arcs: P. Chapelle *et al* 2006;
- α- and γ-modes in high-pressure RF discharges;

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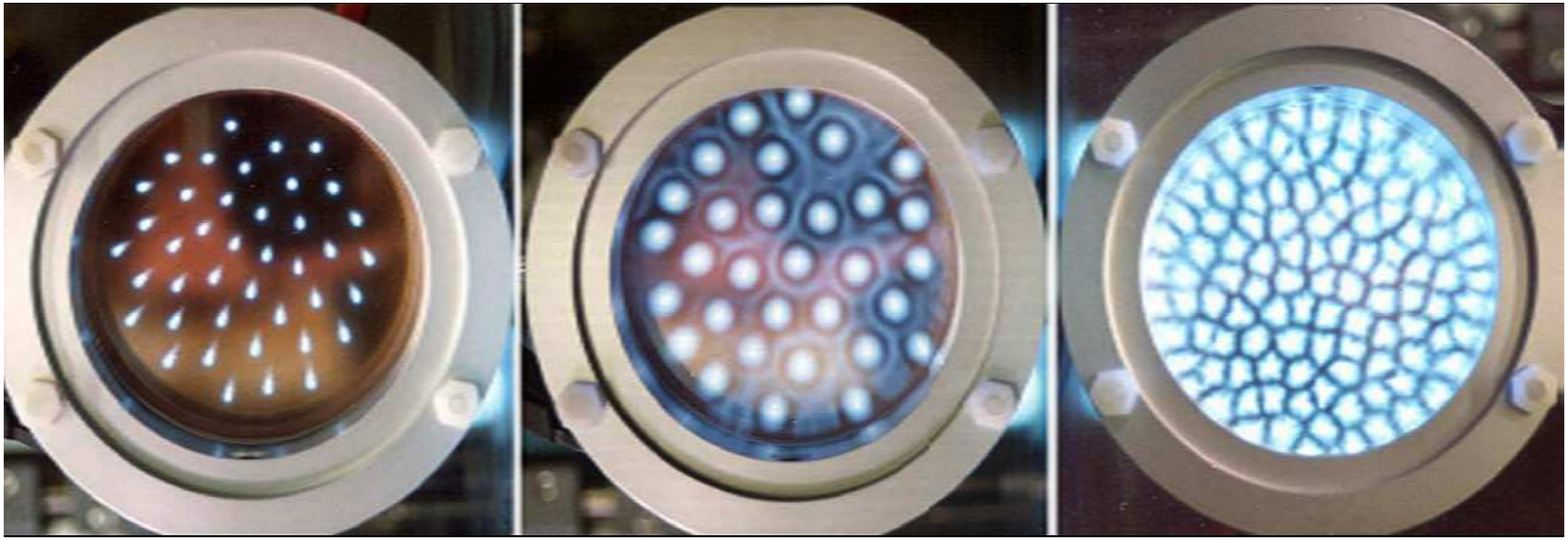
Conclusions

- Regular patterns in barrier discharges



End-on observation of a planar barrier discharge.
5mm gap, Xe/Cl_2, 1 kHz. From U. Kogelschatz 1992.