UNIFIED THEORY OF CATHODE SPOTS IN DC DISCHARGES

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ABSTRACT

Current transfer from high-pressure DC arc plasmas to thermionic cathodes may occur in the diffuse mode, when the current is distributed over the front surface of the cathode in a more or less uniform way, or in the constricted (spot) mode, when most of the current is localized in one or more small areas (cathode spots). A similar phenomenon is observed on cold cathodes of DC glow discharges: current transfer can occur in the abnormal mode, when the current is distributed over the cathode surface in a more or less uniform way, or in the constricted (normal) mode, when only part of the cathode surface is active.

Although physical mechanisms are very different, the overall patterns of the two phenomena are similar. From the point of view of general theoretical physics, both phenomena belong to the class of self-organization phenomena. This allows one to develop a unified treatment of both phenomena. Such treatment is a subject of the present work.

INTRODUCTION

The problem of an adequate theoretical description of different modes of current transfer to cathodes of DC discharges is one of the oldest unsolved problems of gas discharge physics. For quite a long time, theoretical investigation was centered on analysis of the role played by various physical mechanisms in formation of different modes. For example, the role of thermo-field or field emission vs. Schottky-amplified thermionic emission was investigated in connection with constriction of current on hot arc cathodes; the role of diffusion of the ions and the electrons vs. drift, volume ionization, and γ-process was studied in connection with normal and abnormal modes on cold glow cathodes. However, an adequate theoretical description of multiple modes of current transfer to a DC cathode does not necessarily involve essentially different physical mechanisms. This is rather a mathematical question of finding non-unique solutions: an adequate theoretical model of current transfer to a DC cathode must in some cases allow multiple steady-state solutions to exist for the same conditions (in particular, for the same discharge current), different solutions describing different modes of current transfer.

Let us assume that a model has been formulated which describes current transfer to a cathode of a DC discharge. The model comprises nonlinear multidimensional differential equations, corresponding boundary conditions, all relevant transport, kinetic, and material data *etc*. The model admits a 1D solution, $f=f(z)$, and multidimensional solutions, $f=f(x,y,z)$; here x,y,z are Cartesian coordinates, the z-axis being directed normally to the cathode surface. (Note that the existence of a 1D solution implies homogeneity in the transversal directions; in particular, there are no non-uniformities on the cathode surface.) The 1D solution describes a mode with a uniform current distribution along the cathode surface, i.e. the diffuse mode on a hot arc cathode or the abnormal mode on a cold glow cathode. The multidimensional solutions describe modes with a non-uniform current distribution along the cathode surface, i.e., spot modes on an arc cathode or normal modes on a glow cathode.

Thus, the problem of theoretical description of different modes of current transfer amounts to finding multiple solutions to the considered problem, both 1D and multidimensional ones. One could think that, after this has been realized, all the rest is a matter of numerical simulations: modern computers and modern software allow one to find 2D and 3D solutions as a matter of routine. There is, however, a major difficulty in computation of solutions to nonlinear multidimensional problems in cases where these solutions are not unique, which is best expressed as follows: if iterations converge painlessly, then the converged solution is usually not the one that is sought. As far as the problem of current transfer to a cathode of a DC discharge is concerned, this means that enough qualitative information must be available in advance in order for a multidimensional solution to be computed. In the first place, one should know in advance whether the multidimensional solution being sought exists at the current value specified; a question rather difficult to answer by purely numerical means: if iterations have diverged or converged to a 1D solution rather than to the multidimensional solution being sought, one would hardly know whether this is a numerical problem or the multidimensional solution being sought simply does not exist under conditions specified.

Thus, it is appropriate to try to derive qualitative information on spot modes on arc cathodes and on normal modes on glow cathodes in order to clarify the physics relevant, make possible computation of multidimensional solutions describing these modes, and facilitate analysis of computation results. This can be achieved by invoking the theory of self-organization: from the point of view of general theoretical physics, constriction of current on DC cathodes is a phenomenon of self-organization, and trends governing such phenomena are well understood.

GENERAL PATERN OF STEADY-STATE MODES

Let us assume that a 1D solution describing the diffuse mode on an arc cathode or the abnormal mode on a glow cathode has been computed and the corresponding current-voltage characteristic *U(j)* found (here *U* is the near-cathode voltage drop and *j* is the density of electric current from the plasma to the cathode surface). A typical computed current-voltage characteristic is schematically represented by the line AEF in Fig. 1; note that the point A may be, or not, positioned at infinity, depending on a particular model. The current-voltage characteristic is U-shaped, i.e., contains a falling section AE and a section of growth EF, separated by a minimum E. This non-monotony is of primary importance for understanding the current constriction. It is caused by a non-monotonic dependence of the energy flux from the plasma on the local surface temperature in the case of arc cathodes and by the saturation of dependence of the ionization coefficient on the local electric field in the case of glow cathodes.

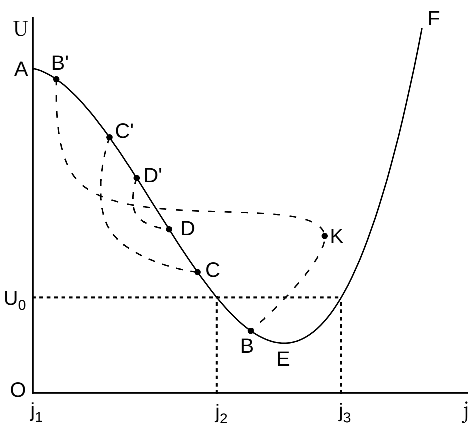

Figure 1. Schematic of a current-voltage characteristic. Solid line: the diffuse mode on an arc cathode and the abnormal mode on a glow cathode. Dashed lines: spot modes on an arc cathode and normal modes on a glow cathode. B, B', C, C', D, D': bifurcation points. K: turning point of the first spot or, respectively, normal mode.

The line AEF is not a full 1D current-voltage characteristic but rather just one branch of it: the full characteristic includes, in addition to this branch, also the branch OA which coincides with the axis of voltages. The latter branch corresponds to the situation where no discharge is present and is described by the trivial solution of the considered problem.

Thus, the full 1D current-voltage characteristic is represented by the line OAEF and is N-shaped. N-shaped characteristics are typical for systems in which self-organization is present. The following hypotheses can be suggested on the basis of general trends typical for such systems. As shown in Fig. 1, there are three 1D solutions at $U=U_0$, the corresponding current densities being designated $j_1=0$, j_2, j_3. The solutions with $j=j_1$ and $j=j_3$ are stable, the one with $j=j_2$ is unstable. One can think also of multidimensional solutions which would describe states in which part of the cathode surface is current-collecting with $j=j_3$ and another part current-free, $j=j_1=0$. In terms of theoretical physics, these are states in which the hot phase, $j=j_3$, co-exists with the cold phase, $j=j_1$. Such co-existence is only possible at a certain value of the near-cathode voltage drop, this value being determined by a condition of solvability of a problem describing an intermediate (transition) region that separates the two phases. (In theoretical-physics terms, this condition may be called Maxwell's construction.)

The above-described multidimensional solutions exist if transversal dimensions of the cathode (dimensions in the directions *x* and *y*) are large enough. On thin cathodes, such solutions exist under another form or do not exits at all.

If self-organized patterns exist, they appear as a result of development of an instability with respect to multidimensional perturbations. If the increment of the instability is real (and positive), then perturbations grow exponentially. If the increment is complex, perturbations grow in an oscillatory way. It is known from the experiment that the appearance of cathode spots is a monotonic process (for example, the luminosity of the cathode surface during the appearance of cathode spots varies in time monotonically). Hence, the increment of the corresponding instability is real. It follows that the change of stability of the 1D solution may occur only through neural stability, i.e., through the zero value of the increment. In other words, there is a point on the 1D CVC where the increment is zero. Perturbations are steady-state at this point. The latter means that at least one multidimensional steady-state solution branches off from (or joins) the 1D solution at this point. Since the unstable one is the falling section of the current-voltage characteristic of the 1D solution, such bifurcations can occur only on this section.

Thus, we come to the conclusion that if multidimensional solutions exist, they branch off from or join the 1D solution, and this branching occurs on the falling section of the current-voltage characteristic of the 1D solution. It should be emphasized that numerical calculations required for finding bifurcation points are one- rather than multidimensional. Bifurcation points associated to each type of multidimensional solutions may exist in pairs; B and B′, C and C′, D and D′ in Fig. 1. If this is the case, then a multidimensional solution which branches off at one of the two points joins at the other one, as is shown by dashed lines in Fig. 1. If the bifurcation analysis has detected no bifurcation points (which happens if the cathode is thin enough), this is an indication that no multidimensional solutions exist under conditions considered; only diffuse mode is possible on an arc cathode and only abnormal mode is possible on a glow cathode. (Note that this conclusion does not apply in cases where only one bifurcation point exists for each type of multidimensional solutions.)

In addition to finding bifurcation points, the bifurcation theory allows one to find asymptotic behavior of multidimensional solutions in the vicinity of bifurcation points. This behavior can be used as an initial approximation for numerical modeling.

Far away from bifurcation points, spots described by multidimensional solutions shrink and occupy only a small part of the cathode surface. One can speak of well-developed spots in such situations. The interaction of a spot with distant parts of the cathode and with other spots becomes weak, i.e., such spots behave as solitary ones.

Thus, ideas of the self-organization theory indeed provide useful information on different modes of current transfer to cathodes of DC discharges, in particular, allow one to establish the pattern of different modes.

STABILITY OF STEADY-STATE MODES

The question arises which of the (multiple) steady-state solutions are stable and can be observed in the experiment.

Some authors have assumed, on the basis of arguments stemming from Steenbeck's principle of minimum voltage (power) for discharges with a fixed current, that a mode with the lowest near-cathode voltage drop is the preferred one. However, Steenbeck's minimum principle bears no relation to fundamental physical principles; although some authors seem to believe that it can be proved by methods of thermodynamics, we have not encountered any such proof in the literature. Equally, Steenbeck's minimum principle bears no relation to the stability theory. In summary, this principle is just an arbitrary assumption and arguments based on this principle are not convincing.

Plausible hypotheses concerning stability of different steady-state solutions have been put forward on the basis of ideas of the self-organization theory. These hypotheses can be summarized as follows. The 1D mode is stable against small perturbations beyond the first bifurcation point (at $I>I_b$, where I is the discharge current and I_b is current corresponding to the first bifurcation point; point B in Fig. 1) and unstable at lower currents (at $I<I_b$). If the cathode is not too thin, the first multidimensional mode branches from the diffuse mode into the range $I>I_b$ in which the 1D mode is stable. Such bifurcations are known as subcritical. By analogy with other problems in which subcritical bifurcations occur, one could expect that the initial section of the first multidimensional mode is unstable. One could expect also that the change of stability of the first multidimensional mode occurs at the turning point (point K in Fig. 1): this mode is stable beyond the turning point and unstable at lower voltages. The second and subsequent multidimensional modes are unstable.

In the framework of these hypotheses, the transition from the diffuse mode on an arc cathode to the spot mode and from the abnormal mode on a glow cathode to the normal mode occurs at $I=I_b$ and the reverse transition occurs at $I=I_t$, where I_t is the value of the discharge current corresponding to the turning point of the first multidimensional mode (point K in Fig. 1). Since $I_b<I_t$, the transitions manifest hysteresis.

RESULTS FOR PARTICULAR MODELS OF CURRENT TRANSFER

According to the above, the following steps are appropriate in order to develop a complete theory of current transfer to cathodes of DC discharges:

a) To formulate a model of plasma-cathode interaction for the particular discharge. While being multidimensional in nature, this model must allow 1D solutions;
b) To find 1D solutions and to perform bifurcation analysis in order to find bifurcation points in which multidimensional solutions branch off from the 1D solution and, eventually, also asymptotic behavior of multidimensional solutions in the vicinity of the bifurcation points;
c) To find multidimensional solutions by means of numerical modeling with the use of information obtained at the preceding step;
d) To extend the theory in order to take into account the fact that distributions of electric current along the surface of a hot arc cathode operating in the diffuse mode and of a cold glow cathode operating in the abnormal mode are not precisely uniform, due to collection of current by the lateral surface of the arc cathode and due to diffusion of the charged particles to the lateral surface of the discharge tube in which the glow discharge is maintained;
e) To investigate stability of steady-state solutions found at the preceding steps.

By now, the steps a) – d) have been successfully completed for cathodes of high-pressure arc discharges, due to efforts developed by a number of workers during the last decade [1-14]. It is remarkable that theoretical advances were interconnected with considerable advances achieved in the same time in the experimental investigation of near-cathode phenomena in high-pressure arc discharges (e.g., [13,15-19]).

The theory is based on the model of nonlinear surface heating, which can be briefly described as follows. A steady-state temperature distribution in the body of a thermionic cathode is considered. The base of the cathode is maintained at a fixed temperature by external cooling and the rest of the cathode surface is in contact with the plasma or the cold gas and is heated or, respectively, cooled. Mathematically, the problem amounts to solving the thermal-conduction equation in the cathode body with a nonlinear boundary condition describing heat exchange with the plasma or with the cold gas.

A whole "zoo" of very diverse steady-state solutions describing different spot modes has been found. The above-described hypotheses, based on ideas of the self-organization theory, have been confirmed. Theoretical results are in agreement with experimental data.

As far as abnormal and normal modes on cold glow cathodes are concerned, only steps a) and b) have been completed [20], on the basis of a fluid model of near-cathode sheath of a glow discharge controlled by diffusion. On the other hand, the interest in this field is still high, in particular, in connection with high-pressure micro-discharges, and one can hope that further progress will follow. Given the above-discussed common features, work in this field will be favored by advances achieved in understanding the diffuse and spot modes on hot arc cathodes.

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