ESCAPE FACTORS FOR THERMIONIC CATHODES

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ABSTRACT

An approximate analytical expression is obtained for the escape factors for thermionically emitting cathodes in atomic gases that is uniformly valid at all values of the reduced electric field. An independent evaluation is performed by means of Monte Carlo simulations. The analytical results for escape factors in neon, helium and mercury are in a good agreement with the results of Monte Carlo simulations, both for reflecting and non-reflecting cathodes.

INTRODUCTION

An appropriate boundary condition describing electron balance at the cathode surface is of fundamental importance to fluid modeling of gas discharges. This boundary condition is conventionally formulated in terms of the escape factor f_{es}; see, e.g., [1]. In order to formulate this boundary condition explicitly, one needs to know the dependence of f_{es} on the electric field E in the near-cathode region.

In [2], an analytical expression for f_{es} has been derived for the range of low values of the reduced electric field where the effect of electric field on the energy relaxation of emitted electrons is minor. In [3], an analytical expression for f_{es} in atomic gases has been derived for the ranges of intermediate to high values of the reduced electric field, where dominating electron energy losses are due to inelastic collisions of electrons with atoms.

It is desirable to develop, on the basis of analytical results [2,3], an analytical expression for f_{es} which would be uniformly valid at all values of the reduced electric field. This task is dealt with in the present communication for the case of thermionic cathodes. The expression derived is used for evaluation of the escape factors in neon, helium and mercury. An independent evaluation of the escape factor is performed by means of Monte Carlo simulations, representing the most accurate technique for studying the influence of different parameters, such as energy distribution function of emitted electrons and reflection of electrons from the electrode, on the back diffusion. A good agreement between the analytical and Monte Carlo results is found.

ANALITICAL FORMULA FOR THE ESCAPE FACTOR IN ATOMIC GASES

In different ranges of electric field values E, the distribution function of emitted electrons and, consequently, the escape factor are governed by different physical mechanisms. Hierarchy of these ranges is governed by the parameter $\rho = (M/2m)^{1/2}\varepsilon_0/\varepsilon_{ex}$ [3], where ε_0 is the average energy with which an electron is emitted, ε_{ex} is the energy of excitation of atoms and m and M are the masses of electrons and atoms, respectively. ρ is typically of the order unity for gases of light atoms with high excitation energy and large for gases of heavy atoms with low excitation energy.

Let us restrict ourselves, for now, to the case $\rho >> 1$. In this case, the dependence of the escape factor f_{es} on E is characterized by three scales of electric field which are, in the increasing order, $\varepsilon_0/e\lambda_u$, $\varepsilon_{ex}/e\lambda_u$, $\varepsilon_0/e\lambda_e$ [3], where λ_e is the mean free path of emitted electrons and $\lambda_u = \lambda_e(M/2m)^{1/2}$ is the length of energy transfer in elastic collisions electron-atom.

In the range $E << \varepsilon_0/e\lambda_u$ the energy gained by electrons on the length scale λ_u is much smaller than ε_0. Hence, the effect of electric field on the energy relaxation of emitted electrons is minor. In particular, the equilibrium electron distribution, which is established due to elastic collisions and holds on distances from the cathode surface much larger than λ_u, is close to the Maxwellian function with the gas temperature. It is natural to term this range the range of low electric fields. A formula for the escape factor for this case was derived in [2] by means of asymptotic analysis of the equation for the isotropic part of the electron distribution function; see Eq. (9) of [2]. In particular, for a thermionically emitting cathode, where the distribution of emitted electrons is a Maxwellian function with the cathode surface temperature T, the latter coinciding with the gas temperature, the escape factor is

$$\chi^{(LF)} = \frac{4eE}{3p(kT)^2}\int_0^\infty \varepsilon e^{-\varepsilon/kT}\, d\varepsilon / Q_m(\varepsilon) \quad (1)$$

where $Q_m(\varepsilon)$ is the transport (momentum-transfer) cross section of collisions of electrons with atoms, ε is the electron kinetic energy, $p = nkT$ is the plasma pressure, and n is the number density of atoms.

Let us consider now the range $\varepsilon_{ex}/e\lambda_u << E << \varepsilon_0/e\lambda_e$. This range will be termed the range of intermediate electric fields. Finding the escape factor in this range amounts to solving the equation describing the isotropic part of the electron distribution function with account of electron energy losses only in inelastic collisions. This solution was found in [3]. In particular, the escape factor for a thermionically emitting cathode is

$$\chi^{(IF)} = \frac{4eE}{3p} \int_0^{\varepsilon_{ex}/kT} e^{-x} dx \Big/ \int_{kTx}^{\varepsilon_{ex}} d\varepsilon \, Q_m(\varepsilon)/\varepsilon \,. \quad (2)$$

One can see from Eqs. (1) and (2) that the escape factor in both ranges of low and intermediate fields is proportional to the electric field. It is natural to try to describe the escape factor in all the ranges from low to intermediate fields by means of the interpolation formula

$$\chi = \frac{\chi^{(LF)} + \zeta E \chi^{(IF)}}{1 + \zeta E}, \quad (3)$$

where ζ is a constant (independent of E) parameter. Obviously, the right-hand side of Eq. (3) represents a weighted average of $\chi^{(LF)}$ and $\chi^{(IF)}$ and the weights are equal at $E = 1/\zeta$. Let us assume that the latter happens in the center of the range $\varepsilon_0/e\lambda_u << E << \varepsilon_{ex}/e\lambda_u$, i.e., at $E = (\varepsilon_0\varepsilon_{ex})^{1/2}/e\lambda_u$. In other words, we set $\zeta = e\lambda_u/(\varepsilon_0\varepsilon_{ex})^{1/2}$ or, equivalently,

$$\zeta = \frac{e}{\sqrt{\varepsilon_0 \varepsilon_{ex}} n \overline{Q}_m} \sqrt{\frac{M}{2m}}, \quad (4)$$

where $\overline{Q}_m$ is a mean value of the transport cross section.

A formula uniformly valid at $E >> \varepsilon_{ex}/e\lambda_u$ was obtained in [3] by means of an asymptotic interpolation (a two-point Padé approximant) between Eq. (2) and the value $f_{es} = 1$ at $E >> \varepsilon_0/e\lambda_e$ and reads

$$f_{es} = \frac{\chi^{(IF)}}{1 + \chi^{(IF)}}. \quad (5)$$

Here $\chi^{(IF)}$ is the escape factor for the range of intermediate electric fields, given in the case of a thermionic cathode by Eq. (2). Replacing in this formula $\chi^{(IF)}$ by χ the escape factor for the ranges from low to intermediate fields given by Eq. (3), one arrives at a formula uniformly valid at all E:

$$f_{es} = \frac{\chi}{1 + \chi}. \quad (6)$$

In what follows, Eq. (6) is used for calculation of escape factors for Ne, He and Hg plasmas. (Note that the particular case of a mercury plasma is of considerable importance for simulation of interaction of thermionic cathodes with high-pressure arc plasmas in high-intensity discharge lamps.) The transport cross sections are taken from [4] (for neon and helium) and [5] (for mercury).

MONTE CARLO SIMULATIONS

In addition to evaluation by means of Eq. (6), the escape factors for Ne, He and Hg plasmas have been evaluated also by means of the Monte Carlo code [6]. The latter is a null collision code for dc fields [7] that has all the features required to model both the relaxed hydrodynamic properties and the non-hydrodynamic development close to electrodes. At the moment of collision, the type of collision for each projectile particle (electron) is determined by a random number. For each particle, a total collision probability can be determined, independent of particle energy and position, as

$$P_t = 1 - \exp(-\nu_{max} dt), \quad (7)$$

where maximum collisional frequency is given by the expression

$$\nu_{max} = n \max_{\varepsilon} (\sigma_t(\varepsilon) v(\varepsilon)). \quad (8)$$

In the above equation, n is a spatially uniform target density, $v(\varepsilon) = (2\varepsilon/m)^{1/2}$ is the incident speed of a particle with energy ε, dt is the time interval and the total cross section $\sigma_t(\varepsilon)$ represents the sum over all processes j:

$$\sigma_t(\varepsilon) = \sum_j \sigma_j(\varepsilon). \quad (9)$$

The number of projectile particles dN taking part in collisions at each time step is given by the total collision probability

$$dN = P_t N, \quad (10)$$

where N is the total number of projectile particles.

In order to implement effects of back diffusion of electrons, a part is added that checks if electron goes back to the cathode after collision or continues traveling to the anode. Furthermore, the reflection of electrons from the cathode is considered, bearing in mind that electrons are reflected from the cathode surface without any energy loss. More precisely, the code follows individual electrons released from the cathode until they reach either anode or cathode. When an electron hits the cathode it may be absorbed, or it may be reflected with the given energy and angular distribution.

The code has been applied to model electron transport in argon [8], nitrogen [9], neon and xenon [10] and many other gases and has been also used to derive the cross sections for electron excitation. The code has been tested extensively against other codes and numerical techniques and was found to produce transport data limited in accuracy only by the accuracy of cross sections and statistical scatter. When the code was modified to calculate escape (back diffusion) coefficients a special care was taken to include reflection from the cathode.

In this work, the code is used for calculation of escape factors for thermionic emission. The distribution of emitted electrons was assumed Maxwellian in these calculations. Note that calculations for both Maxwellian and monoenergetic distributions of emitted electrons have been reported in [8]. It was found in those calculations that results are very sensitive to the choice of the initial energy and its distribution. When the initial energy distribution is broad there is a large number of electrons with energies close to zero and they cannot return to the cathode. The dependence of the escape factor on the initial (monoenergetic) energy is quite nonlinear and thus for low mean initial energies the results are quite sensitive to the choice of distribution. In the present work, calculations of escape factors by means of the Monte Carlo code have been performed using the sets of electron-atom collision cross sections for neon [9] and for helium [4]. Both sets are based on the data [4,10,11] which have been completed by adding excitation cross sections and extrapolating the available cross sections to higher energies. However, for moderate energies that are covered here the cross section sets should be fully compatible with the recommended cross sections of A.V. Phelps [4]. In particular, the cross section sets were tested to reproduce the low energy electron transport data.

RESULTS

In figures 1-3, results for the escape factors given by Eq. (6) and by the Monte Carlo code are given for thermionic emission into neon, helium and mercury plasmas from non-reflecting cathodes for two values of the cathode temperature, corresponding to the mean energies of emitted electrons ε_0 = 0.2eV and 0.6eV (note that $\varepsilon_0 = (3/2)kT$). One can see that for all three gases the dependence of f_{es} on E, predicted by the interpolation formula for the range between the low-field and intermediate-field regions is weaker than linear. The low-field values, $\chi^{(LF)}$, and the values calculated with the use Eq. (5) are also shown.

Note that expression (4) for the interpolation parameter ζ contains a mean transport cross section $\overline{Q}_m$ which is not uniquely defined and may be shifted within certain limits. It is natural to choose such value of this parameter that ensures the best fit of results given by Eq. (6) into the Monte Carlo data.

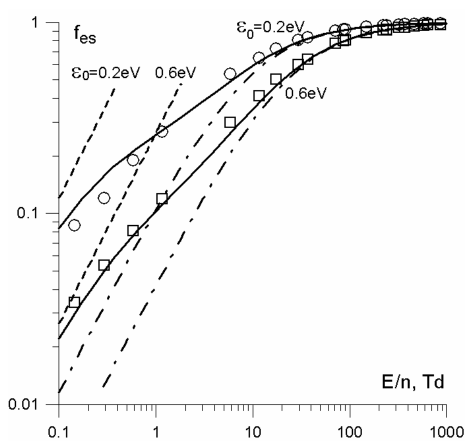

Figure 1. The escape factor in Ne versus the reduced electric field for emitted electrons with the mean energies 0.2 and 0.6 eV. Points - Monte Carlo data; solid lines - Eq. (6), dashed - (1), dot-dashed - (5).

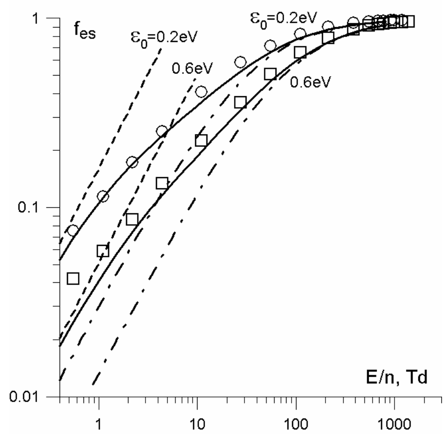

Figure 2. The escape factor in He versus the reduced electric field for emitted electrons with the mean energies 0.2 and 0.6 eV. Points - Monte Carlo data; solid lines - Eq. (6), dashed - (1), dot-dashed - (5).

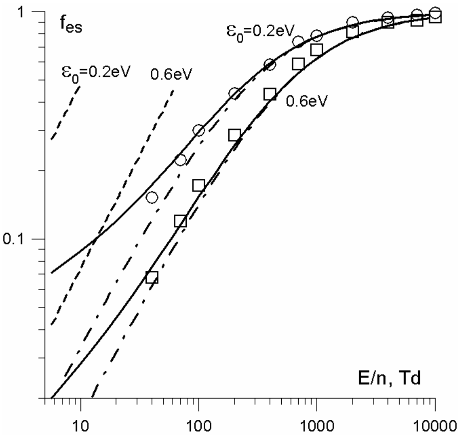

Figure 3. The escape factor in Hg versus the reduced electric field for emitted electrons with the mean energies 0.2 and 0.6 eV. Points - Monte Carlo data; solid lines - Eq. (6), dashed - (1), dot-dashed - (5).

The data shown in figures 1-3 correspond to $\overline{Q}_m$ values equal to 2×10^{-20} m² for Ne, 6×10^{-20} m² for He and 7×10^{-19} m² for Hg. It is seen that the interpolation formula (6) gives, for all gases, values of f_{es} that are in a reasonable agreement with the Monte Carlo data.

The escape factors for reflecting cathodes may be evaluated by means of the approximate formula [3]

$$f_{es}^{(R)} = f_{es}[1 + R(1\text{-}f_{es})], \qquad (11)$$

where R is the reflection coefficient and f_{es} is the escape factor without reflection given by Eq. (6). In figures 4-6 the escape factors are shown for thermionic emission into neon, helium and mercury plasmas from reflecting cathodes with R = 0.6, calculated by means of Eq. (11) for ε_0 = 0.2eV . For comparison, values of f_{es} at R = 0 are also shown. In the same figures the results are presented also of Monte Carlo simulation for reflecting and non-reflecting cathodes. It is seen that the approximate account of reflection according to Eq. (11) results in f_{es} values rather close to the Monte Carlo data.

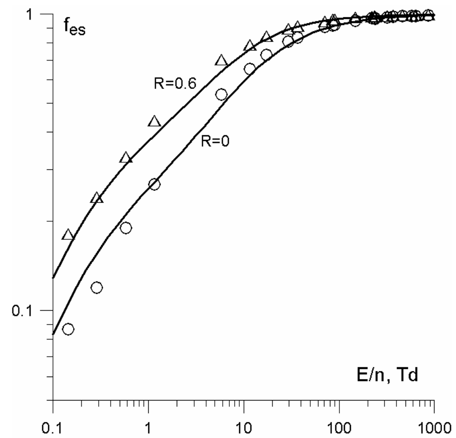

Figure 4. The escape factor in Ne versus the reduced electric field for emitted electrons with the mean energy 0.2eV, at two values of the reflection coefficient, 0 and 0.6. Points - Monte Carlo data; lines - Eq. (11).

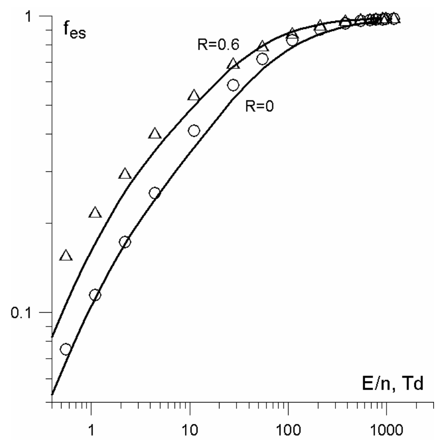

Figure 5. The escape factor in He versus the reduced electric field for emitted electrons with the mean energy 0.2eV, at two values of the reflection coefficient, 0 and 0.6. Points - Monte Carlo data; lines - Eq. (11).

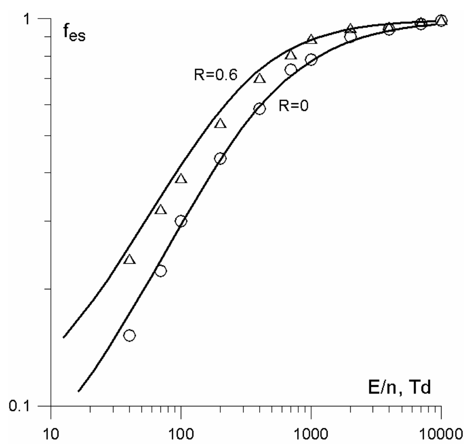

Figure 6. The escape factor in Hg versus the reduced electric field for emitted electrons with the mean energy 0.2eV, at two values of the reflection coefficient, 0 and 0.6. Points - Monte Carlo data; lines - Eq. (11).

CONCLUSION

An analytical expression has been derived for escape factors for thermionically emitting cathodes in atomic plasmas, which is uniformly valid at all values of the reduced electric field. Results for the escape factors in neon, helium and mercury are given. An independent evaluation of the escape factor is performed by means of Monte Carlo simulations. A good agreement is found between the analytical data and Monte Carlo results.

The results obtained may be used for a rapid evaluation of escape factors for thermionic cathodes in atomic plasmas, in particular, for cathodes of high-intensity discharge lamps.

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