Bifurcations of current transfer through a collisional sheath and self-organization on glow cathodes

M.S. Benilov

Departamento de Física, Universidade da Madeira, Largo do Município, 9000 Funchal, Portugal

Bifurcation analysis is performed of a boundary-value problem describing the near-cathode region of a high-pressure glow discharge. The mechanisms taken into account are drift of the ions and the electrons, volume ionization and losses, and secondary electron emission. Calculation results are given for parallel-plate geometry and for a near-cathode space-charge sheath adjacent to a uniform plasma column. In both configurations, bifurcations are detected on the growing section of the current density-voltage characteristic, corresponding to the abnormal discharge, and the bifurcating solutions describe patterns with many spots. These results conform to recently published observations of self-organization on molybdenum or tungsten cathodes of DC microdischarges in high-pressure xenon plasmas.

1. Introduction

Existence of different modes of current transfer to electrodes in DC gas discharges seems to be rather a rule than exception. As far as cathodes of glow discharges are concerned, it has been known for many decades that current transfer occurs in the abnormal mode or in the mode with a normal spot (e.g., [1]); recently, also modes with multiple spots have been observed [2,3]. Diffuse and spot modes can occur on cathodes of high-pressure arc discharges; e.g., [4] and references therein. Diffuse, constricted, and multiple-spot modes can occur on anodes of high-pressure arc discharges; e.g., [5,6].

The ultimate goal of a theory of multiple modes of steady-state current transfer is the capability of predicting what modes are possible for a given discharge current and which of these modes are stable. At present, this goal has been achieved only in the theory of cathodes of high-pressure arc discharges; see [7] and references therein. The theory is much less advanced for cathodes of glow discharges; the only aspect that is developed relatively well is numerical modelling of normal spots; e.g., [8,9]. As far as multiple-spot modes on glow cathodes are concerned, opinions diverge even on such basic question as what mechanisms are responsible for formation of multiple spots [2,3,10].

The development of a theory of multiple modes of current transfer to cathodes of high-pressure arc discharges was initiated by an application of the bifurcation analysis [11]. Bifurcation analysis is useful also in the theory of multiple modes on glow cathodes [12]. In the present paper, a bifurcation analysis is performed of a fluid system of equations describing current transfer to cathodes of high-pressure glow discharges. Two discharge configurations are treated, a discharge between parallel plates and a collision-dominated sheath adjacent to a uniform positive plasma column. Calculations are performed for a high-pressure Xe plasma. In both discharge configurations, bifurcations have been detected in the abnormal mode of glow discharge. As the current density is decreased in the abnormal mode, the first bifurcation point to be encountered is associated with a pattern with many spots, in accord to what is observed in the experiment [2,3].

2. The approach

Let us consider a high-pressure DC glow discharge in a discharge vessel in the form of a right cylinder, not necessarily circular; see figure 1. The origin of Cartesian coordinates x, y, z is on the cathode surface and the z-axis is perpendicular to the cathode surface, i.e., parallel to the lateral surface of the discharge vessel.

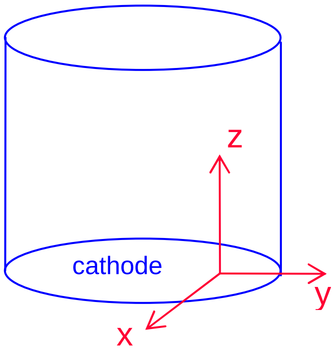


Figure 1. Geometry of the problem.

In the abnormal mode, where the discharge current is high enough, it is distributed over the cathode surface more or less uniformly and the distribution of discharge parameters in the near-cathode region is 1D, $f = f(z)$. Modes with spots are associated with multidimensional distributions of parameters, $f = f(x, y, z)$. Hence, an adequate theoretical model must admit different kinds of steady-state solutions, a 1D one and multidimensional ones. Furthermore, the 1D solution should exist at all current values, although at low currents it is unstable and does not realize. Hence, an adequate theoretical model must admit multiple steady-state solutions to exist for the same discharge current, at least in a certain current range.

It happens frequently in problems with multiple solutions of different symmetries that two solutions become exactly identical at a certain value of a control parameter; a phenomenon called branching of solutions, or bifurcation. There are reasons to believe that this occurs also in the problem considered here, i.e., multidimensional solutions describing spot modes branch off from the 1D solution describing the abnormal mode (or, more precisely, the abnormal mode and the unstable mode corresponding to the falling section of the current density-voltage characteristic, or CDVC). Values of the current density at which this branching occurs (bifurcation points) represent points of neutral stability of the 1D solution against multidimensional perturbations, and can be determined by means of the well-known formalism of the linear theory of stability. Thus, in order to answer the question whether a theoretical model of a glow discharge describes steady-state patterns of a given kind, one can try to find bifurcation points at which multidimensional solutions branch off from the 1D solution. The answer is positive if such points exist and the multidimensional solutions in the vicinity of these points are qualitatively similar to the patterns in question.

3. Results

It is found, similarly to how it has been done in [11,12], that the dependence of multidimensional solutions on x and y is described in the vicinity of bifurcation points by eigenfunctions Φ of the 2D Helmholtz equation

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + k^2\Phi = 0, \qquad (1)$$

which should be solved in the cross section of the discharge vessel with the homogeneous Neumann boundary conditions. Note that the eigenvalue k may be interpreted as a wave number characterizing variation of the bifurcating spot mode along the cathode.

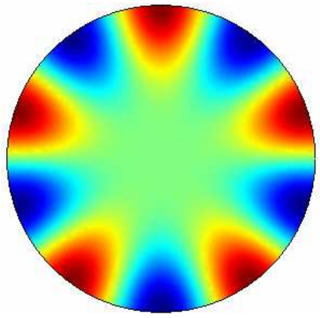
The Neumann eigenvalue problem for equation (1) may be solved analytically for discharge vessels with cross sections of many simple shapes, in particular, for a vessel with a circular cross section:

$$\Phi = J_\nu(kr)\,\cos\nu\theta, \quad k = j'_{\nu,s}/R. \qquad (2)$$

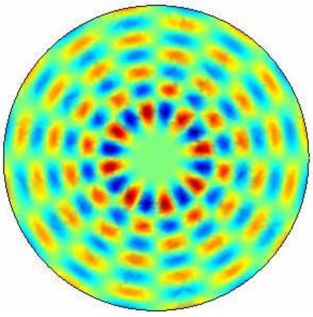
Here (r,θ,z) are cylindrical coordinates with the origin at the center of the cathode, $J_\nu(x)$ is the Bessel function of the first kind of order ν, $\nu = 0,1,2,\ldots$, $j'_{\nu,s}$ is a s-th zero of the derivative of $J_\nu(x)$, $s = 1,2,3,\ldots$, and R is the radius of the vessel. As an example, two of functions (2) are illustrated in figure 2. One can see that distributions described by these functions are not qualitatively different from patterns observed in the experiments [2,3]. Note that some of the observed patterns comprise a spot at the center, which is not described by the functions (2). Such patterns can appear through a bifurcation from axially symmetric spot modes; see, e.g., figure 9 in [13].

Calculation of bifurcation points requires a numerical solution of a 1D eigenvalue problem. In the present work, this was done by means of a fourth-order finite difference scheme [14]. The simplest self-consistent model of a glow discharge was used, which comprises equations of continuity of the ions of one species and of the electrons written with account of drift of the charged particles, ionization, recombination and volume losses due to diffusion, and the Poisson equation. Dissociative recombination was treated as a dominating recombination mechanism, the secondary electron emission coefficient was assumed constant (independent of the electric field at the cathode surface).

Examples of the calculated relationship between the value of the current density j and the normalized wave number of the spot mode that branches off from the abnormal mode at this value of j is shown in figures 3 and 4. The normalization factor Δ is defined as the ratio of the discharge voltage to the electric field on the cathode and has the meaning of a scale of thickness of the near-cathode space-charge sheath. Also shown are CDVC's of the discharge.



(a) $\nu = 5$, $s = 1$.



(b) $\nu = 8$, $s = 7$.

Figure 2. Emerging patterns on a circular cathode, equation (2).

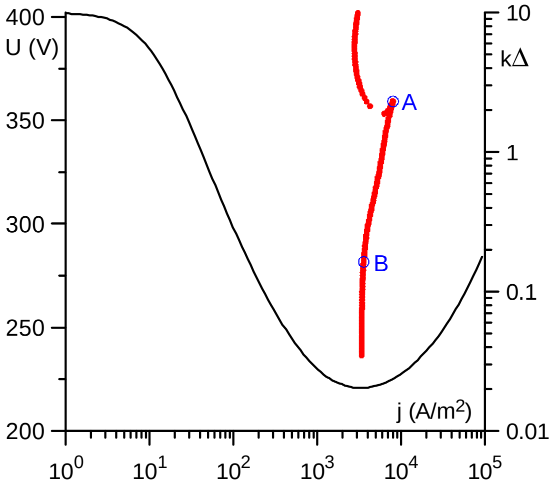


Figure 3. Points: the normalized wave number of the spot mode as a function of the current density at which this mode branches off from the abnormal mode. Line: current density-voltage characteristic. Discharge in xenon at the pressure of 150 Torr. Plane-parallel geometry, the inter-electrode gap is 0.5 mm.

Both figures 3 and 4 refer to xenon. Calculations shown in figure 3 have been performed for the parallel-plane discharge configuration, figure 4 refers to a collision-dominated sheath adjacent to a uniform positive plasma column. The secondary electron emission in the calculations was assumed equal to 0.03. The diffusion losses have been neglected in the calculations shown in figure 3. In the calculations shown in figure 4, the diffusion losses have been estimated in terms of the radius R of the discharge vessel as described in [1].

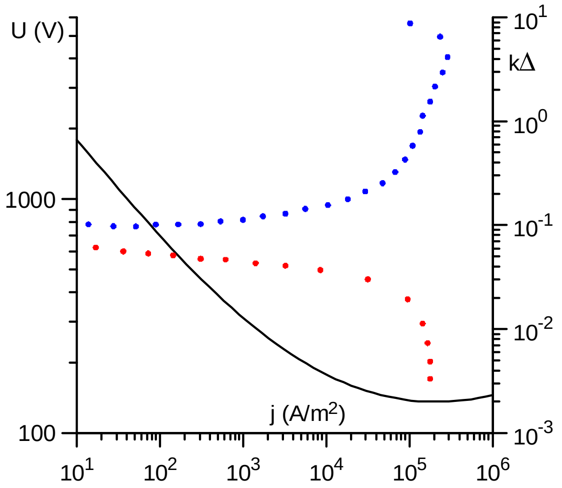


Figure 4. Points: the normalized wave number of the spot mode as a function of the current density at which this mode branches off from the abnormal mode. Line: current density-voltage characteristic. Discharge in xenon at the pressure of 760 Torr, R = 0.4 mm, collision-dominated near-cathode space-charge sheath.

In the limiting case of long wavelengths, $k\Delta << 1$, the bifurcation points tend to the point of minimum of the CDVC as they should. At very short wavelengths, $k\Delta \geq 10$, the model of mobility-dominated space-charge sheath looses its validity. [This model is valid provided that the time of drift of the electrons across the space-charge sheath, $\Delta^2 / \mu_e U$, is much smaller than the time of diffusion in the direction along the cathode, $1/D_e k^2$, and this condition holds at $k\Delta << \sqrt{U/T_e} \sim 10$, where μ_e, D_e, and T_e are the mobility, diffusion coefficient, and temperature of the electrons.]

The point designated A in figure 3, which is the right-most point of the curve $k(j)$, is of particular importance: the abnormal mode is stable at $j > j_A$

and unstable at $j < j_A$. For comparison, also shown in figure 3 is a point B at which the mode with one spot at the edge of the cathode branches off in the case $R = 0.375\,\mathrm{mm}$. The value of k corresponding to the point A, $k_A \approx 8.7\times10^4\,\mathrm{m}^{-1}$, substantially exceeds $k_B \approx 4.9\times10^3\,\mathrm{m}^{-1}$, and modes that branch off in the vicinity of A comprise many spots: the mode with 30 spots at the edge of the cathode, $\nu = 30$, $s = 1$, branches off at $k \approx 8.7\times10^4\,\mathrm{m}^{-1}$; the mode with three concentric layers of 20 spots each, $\nu = 20$, $s = 3$, branches off at $k \approx 8.5\times10^4\,\mathrm{m}^{-1}$; the pattern shown in figure 2b branches off at $k \approx 8.3\times10^4\,\mathrm{m}^{-1}$ *etc*. One can expect therefore that the mode that occurs at $j < j_A$ is a one with many spots, in accord to what is observed in the experiment [2,3].

Data shown in figure 4 are more involved. There are two disconnected families of bifurcation points. The first family occurs in the range $k\Delta \leq 0.1$, the second family occurs in the range $k\Delta \geq 0.1$. In this case, too, the right-most point of the curve $k(j)$ belongs to the abnormal branch of the CDVC and corresponds to a pattern with many spots.

4. Conclusions

Bifurcation analysis is performed of a fluid system of equations describing current transfer to cathodes of high-pressure glow discharges. Two discharge configurations are treated, a discharge between parallel plates and a collision-dominated sheath adjacent to a uniform positive plasma column. Calculations are performed for a high-pressure Xe plasma. In both discharge configurations, bifurcations have been detected in the abnormal mode of glow discharge. As the current density is decreased in the abnormal mode, the first bifurcation point to be encountered is associated with a pattern with many spots, in accord to what is observed in the experiment [2,3].

The question of theoretical description of steady-state spot modes on glow cathodes is far from closed: a complete 3D simulation of different spot modes in the whole range of existence of each mode and an investigation of their stability are needed. Bifurcation analysis performed in this work brings important qualitative information and represents a useful starting point for such simulations: it indicates in which range of parameters each mode should be sought.

Acknowledgments The work was performed within activities of the project POCI/FIS/60526/2004 of FCT, POCI 2010 and FEDER and of the project *Centro de Ciências Matemáticas* of FCT, POCTI-219 and FEDER.

5. References

[1] Yu.P. Raizer (1991) *Gas Discharge Physics* (Springer, Berlin).

[2] K.H. Schoenbach, M. Moselhy, and W. Shi (2004) Plasma Sources Sci. Technol. **13** 177.

[3] N. Takano and K.H. Schoenbach (2006) Plasma Sources Sci. Technol. **15** S109.

[4] S. Lichtenberg, D. Nandelstädt, L. Dabringhausen, M. Redwitz, J. Luhmann, and J. Mentel (2002) J. Phys. D: Appl. Phys. **35** 1648.

[5] S. M. Shkolnik (2000) in *Encyclopaedia of Low-Temperature Plasmas*, edited by V. F. Fortov (Nauka, Moscow) 147, in Russian.

[6] F.G. Baksht, G.A. Dyuzhev, N.K. Mitrofanov, and S.M. Shkol'nik (1997) Tech. Phys. **42** 35.

[7] M.S. Benilov (2007) J. Phys. D: Appl. Phys. **40** 1376.

[8] R.R. Arslanbekov and V.I. Kolobov (2003) J. Phys. D: Appl. Phys. **36** 2986.

[9] S.T. Surzhikov (2005) High Temp. **43** 825.

[10] M.S. Benilov (2007) Plasma Sources Sci. Technol. **17**, in press.

[11] M.S. Benilov (1998) Phys. Rev. E **58** 6480.

[12] M.S. Benilov (1988) Sov. Phys. - Tech. Phys. **33** 1267.

[13] M.S. Benilov, M. Carpaij, and M.D. Cunha (2006) J. Phys. D: Appl. Phys. **39** 2124.

[14] I.V. Petukhov (1964) in *Methods of Numerical Solution of Differential and Integral Equations and Quadrature Formulas (Supplement to Journal of Computational Mathematics and Mathematical Physics)* (Nauka, Moscow) 304.