Numerical investigation of stability of steady-state current transfer to thermionic cathodes

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Spectra of perturbations of steady-state current transfer to thermionic cathodes of a high-pressure argon arc have been computed in the framework of the model of nonlinear surface heating. The following pattern of stability has been established for a current-controlled arc on a cylindrical cathode on the basis of the numerical and asymptotic results: the diffuse mode is stable beyond the first bifurcation point and unstable at lower currents; steady-state modes with more than one spot are unstable; the steady-state axially symmetric mode with a spot at the center of the front surface of the cathode is unstable; the 3D steady-state mode with a spot at the edge is unstable between the bifurcation point and the turning point and stable beyond the turning point; the transition between the latter mode and the diffuse mode is non-stationary and accompanied by hysteresis. Numerical results are in agreement with trends observed in the experiment.

1. Introduction

Interaction of high-pressure arc plasmas with thermionic cathodes is a challenging issue of high scientific interest and technological importance. In spite of many decades of research, a self-consistent theory and modelling methods have started to emerge only recently. Still, some important questions are far from being answered. Stability of different modes of steady-state current transfer from high-pressure arc plasmas to thermionic cathodes is one of such questions.

The present work is concerned with a numerical investigation of stability, with the aim to establish a complete pattern of stability of all steady-state modes on axially symmetric cathodes. Another goal of the present work is to obtain quantitative data on spectrum of perturbations under conditions of experimental interest. Apart from being of interest by itself, such data will allow one to resolve the contradiction between conclusions of the analytical treatment in the preceding work [1] and the pattern of stability of the diffuse and first spot modes that has been proposed in the work [2] and is seemingly supported by the experiment.

2. The model

Stability in the present work is investigated by means of the model of nonlinear surface heating, which has been by now widely recognized as an adequate tool of simulation of interaction of high-pressure arc plasmas with thermionic cathodes. The temperature distribution inside the cathode is governed by the non-stationary equation of heat conduction:

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot \left(\kappa \cdot \nabla T\right), \tag{1}$$

where κ, ρ, and c_p are thermal conductivity, density, and specific heat of the cathode material (known functions of the temperature T) and t is time. Joule heat production inside the cathode is neglected.

The base Γ_c of the cathode is maintained at a fixed temperature T_c by external cooling. The rest of the cathode surface, Γ_h, is in contact with the plasma or the cold gas and exchanges energy with it. The boundary conditions read

$$\Gamma_c : T = T_c; \qquad \Gamma_{\mathrm{h}} : \kappa \frac{\partial T}{\partial n} = q(T,U). \tag{2}$$

Here n is the direction locally orthogonal to the cathode surface and oriented outward and q is the density of the energy flux to the cathode surface from the arc plasma or the cold gas (a known function of the local cathode temperature T and the voltage drop U across the near-cathode layer.)

In the framework of the conventional formalism of the linear stability theory, a solution to the problem (1), (2) is sought as sum of a steady-state solution and a small perturbation with the exponential time dependence. The perturbation obeys a linear eigenvalue problem with λ the growth increment playing the role of eigenparameter. The eigenvalue problem is solved numerically with the use of the commercial software COMSOL Multiphysics, version 3.3.

3. Spectra of different modes of steady-state current transfer

In all the simulations performed, all the spectra turned out to be real, i.e., included only real values of λ. This result conforms to the well-known experimental fact that transitions between the diffuse and spots modes are always monotonic, i.e., occur without oscillations.

Numerical calculations reported in this work have been performed for tungsten cathode and argon plasma. Data on κ, $q(T,U)$ and $j(T,U)$ are the same as in [3], data on c_p have been taken from [4].

Results given in this section refer to a cylindrical cathode of the radius R = 2 mm and height h = 10 mm. The plasma pressure is set equal to 1 bar.

Steady-state modes stability of which has been investigated are shown in figure 1. This figure has been taken from the work [3]; index 0 here and further refers to steady-state values of respective quantities. The open circles represent points of minimum of current-voltage characteristics (CVC's) of axially symmetric modes; corresponding values of the arc current will be designated $I_{\min}$. The full circles represent bifurcation points, i.e., points at which a 3D spot mode becomes exactly identical with an axially symmetric mode. Number v characterizing each bifurcation point represents the number of spots at the edge of the front surface of the cathode in the 3D mode that branches off at this point. Value of the arc current corresponding to a bifurcation point with a given v will be designated I_v. Also shown in figure 1 are turning points (squares).

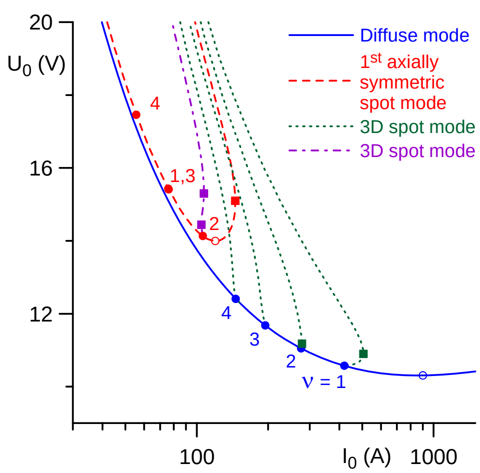

Figure 1. CVC's of different modes of steady-state current transfer.

3.1 Axially symmetric modes

Since steady-state temperature distributions associated with this kind of modes are axially symmetric, perturbations of such modes are harmonic in the azimuthal angle φ, i.e., proportional to $\cos n\varphi$, $n = 0,1,2,\ldots$. Therefore, one can associate each perturbation mode with the corresponding value of n. Perturbations with $n = 0$ are axially symmetric; those with $n > 0$ are 3D.

Results of calculations of stability of the diffuse mode are shown in figure 2. One can see that all modes of perturbations have negative increments on the rising section of the CVC of the diffuse mode (i.e., in the current range $I_0 > I_{\min}$). Hence, this section is stable.

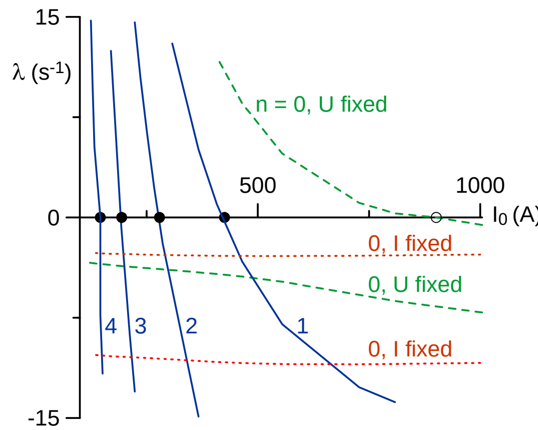

Figure 2. Increments of different modes of perturbations of the diffuse mode. Solid: 3D perturbations. Dashed: axially symmetric perturbations, near-cathode region with a fixed voltage. Dotted: axially symmetric perturbations, current-controlled arc.

There is a mode of axially symmetric perturbations with a positive increment on the falling section of the CVC, i.e., in the range $I_0 < I_{\min}$. All the other axially symmetric perturbation modes remain stable at all currents. At $I_0 < I_1$, another mode of perturbations becomes unstable. The perturbations of this mode are 3D and their dependence on φ is described by the factor cosφ; the first 3D perturbation mode. A perturbation mode that becomes unstable next, at $I_0 < I_2$, is proportional to cos2φ; the second 3D mode. This behavior repeats in the subsequent bifurcation points.

The above results refer to a near-cathode region with a fixed voltage. 3D perturbations are not affected by external resistance, but axially symmetric perturbations are. Calculations for a current-controlled arc show that all the axially symmetric perturbation modes are stable at all currents; see dotted lines in figure 2.

Results on stability of the first axially symmetric spot mode are as follows. (Note that these results refer to a current-controlled arc, as well as all the other results that will follow.) There is a mode of axially symmetric perturbations, n = 0, that is unstable on the low-voltage branch of the first axially symmetric spot mode. On passing through the turning point, this perturbation mode changes is stability and is stable on the high-voltage branch. The other modes of axially symmetric perturbations are always stable. There are modes of 3D perturbations with n = 1,2,3,.., each of which is neutrally stable at the corresponding bifurcation point $I_0 = I_n$, is unstable on the low-voltage branch at lower currents, $I_0 < I_n$, and is stable on the low-voltage branch at higher currents and on the high-voltage branch. There is a mode of 3D perturbations with n = 1 that is unstable at all currents. Because of the latter mode, the first axially symmetric steady-state spot mode is unstable in the whole range of its existence.

3.2 3D spot modes

All perturbations of 3D steady-state spot modes are 3D and their azimuthal dependence is no longer harmonic, therefore the classification of perturbations in terms of n employed in the preceding section is not applicable. However, one can extend this classification using the fact that the "initial" state of each 3D steady-state spot mode, $I_0 = I_\nu$, being a bifurcation point, is axially symmetric, hence perturbations of this state are harmonic. In the following, a mode of perturbations of a 3D steady-state spot mode that is proportional to $\cos n\varphi$ at $I_0 = I_\nu$ will be associated with this value of n.

Increments of perturbations of the steady-state mode with a spot at the edge of the front surface of the cathode are shown in figure 3. This mode branches off from the diffuse mode at a bifurcation point ν = 1 (see figure 1) and possesses a turning point. The dashed lines in figure 3 represent values of increments on that section of the steady-state spot mode that is comprised between the initial state and the turning point; solid lines represent values of increments on the section beyond the turning point. One can see that there is a mode of perturbations with n = 1 that is neutrally stable at the initial state and at the turning point; unstable between the initial state and the turning point; stable beyond the turning point. Perturbations of all the other modes are stable in the whole range of existence of the steady-state mode in question. The 3D steady-state mode with two spots at the edge possesses one turning point as well. There is a mode of perturbations with n = 2 that is neutrally stable at the initial state and at the turning point; unstable between the bifurcation mode and the turning point; stable beyond the turning point. There is a mode of perturbations with n = 1 that is unstable in the whole range of existence of the steady-state mode in question. Perturbations of all the other modes are stable in the whole range of existence.

The 3D steady-state modes with three or four spots at the edge possess no turning points. There is a mode of perturbations with n = 3 (or, respectively, with n = 4) that is neutrally stable at the initial state and stable outside this state. There are two (three) modes of perturbations, one of these modes with n = 1 and the other one with n = 2 (or, respectively, with n = 1, n = 2, and n = 3), that are unstable in the whole range of existence of the steady-state spot mode being considered. Perturbations of all the other modes are always stable.

The mode with two spots at the edge of the front surface of the cathode and a spot at the center possesses two turning points, at arc currents I_0 = 104.5 A and I_0 = 107.2 A. There is a mode of perturbations with n = 2 that is stable between the initial state and the first turning point; unstable between the first and second turning points, stable beyond the second turning point. There is a mode of perturbations with n = 1 that is stable at the initial state and in its vicinity and then turns unstable, the change of stability occurring at I_0 = 105.3 A, i.e., outside the turning points. There are two modes of perturbations, one of these modes with n = 0 and the other with n = 1, that are unstable in the whole range of existence. Perturbations of all the other modes are stable in the whole range of existence.

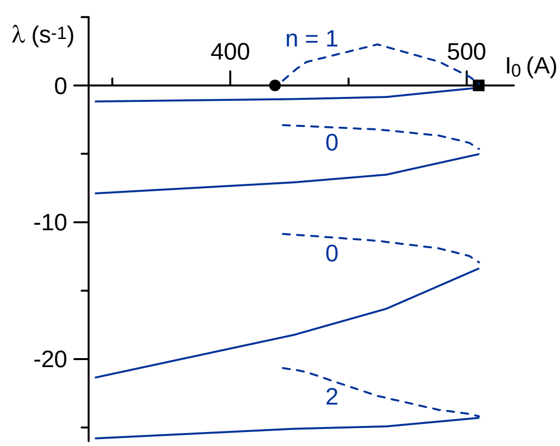

Figure 3. Increments of different modes of perturbations of the steady-state mode with a spot at the edge of the front surface of the cathode.

In summary, the above numerical results show that the only 3D steady-state mode that may be stable is the mode with a spot at the edge. In the following, this mode will be referred to as the first spot mode. This mode is unstable between the bifurcation point and the turning point and stable beyond the turning point. The diffuse mode is stable beyond the first bifurcation point. A transition between these two modes is non-stationary and accompanied by hysteresis.

4. Stability under conditions of experimental interest

In figure 4, stability of modes of DC current transfer to an arc cathode under conditions of the experiment [2] is illustrated. The CVC's of the diffuse mode and of the first spot mode plotted in this figure have been taken from [3]. Bifurcation points positioned on the diffuse mode are located at U_0 well in excess of 100 V and are not seen on the graph. For several steady states which are depicted by points, values of inverse of the biggest increment of perturbations of this state are indicated. Each of these values may be interpreted as time of disruption of the corresponding steady state, if it is positive, or time of decay of perturbations, if it is negative.

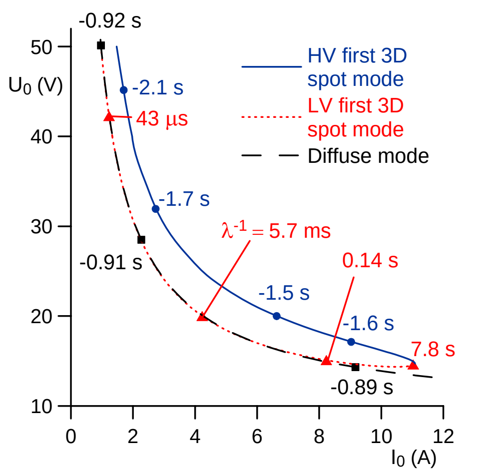

Figure 4. CVC of different modes. R = 0.75 mm, h = 20 mm, the argon pressure 2.6 bar. Numbers: time of development or decay of perturbations.

One can see that $\lambda < 0$ on the diffuse mode and on the high-voltage branch of the first spot mode and $\lambda > 0$ on the low-voltage branch, in accord to the above-described general pattern.

The experiment is usually performed in a limited current and voltage range, say $I_0 \leq 10$ A and $U_0 \leq 100$ V. Data shown in figure 4 indicate that two stable modes exist in the whole range of conditions of such experiment. Hence, no reproducible transition between diffuse and spot modes can occur in a typical quasi-stationary experiment: if the experiment is well-controlled and quasi-stationary, a mode which has occurred immediately after the ignition of the discharge will be maintained during the whole experimental run; if a mode change is systematically observed in such experiment under quasi-stationary conditions, it means that the experiment is not well-controlled. This conclusion conforms to the general trend that the transition between the diffuse and spot modes is difficult to reproduce in the experiment.

In Ref. [2], the low- and high-voltage branches were calculated in the framework of the model of nonlinear surface heating and the assumption was forwarded that it is the low-voltage branch that occurs in the experiment. However, both the numerical results of the present work and the analytical theory [1] indicate that the low-voltage branch is unstable, and this conclusion has been obtained in the framework of the same model of nonlinear surface heating which has been used in the modelling [2]. Hence, the comparison between simulations and experimental results on the spot mode performed in [2] must be revisited.

5. Conclusions

A general pattern of stability of the different modes of current transfer has been established. This pattern conforms to trends observed in the experiment: the diffuse-spot transition on arc cathodes is a monotonic process; patterns with more than one spot are not normally observed; the diffuse mode is observed at high currents and the mode with a spot at the edge of the front surface of the cathode at low currents; the transition between the diffuse mode and the mode with a spot at the edge is non-stationary and is accompanied by hysteresis; this transition is difficult to be reproduced.

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