Modelling of near-cathode layers in high-pressure arc discharges

N.A. Almeida[1], M.S. Benilov[1], G.V. Naidis[2]

[1]Departamento de Física, Universidade da Madeira, 9000 Funchal, Portugal
[2]Institute for High Temperatures, Russian Academy of Sciences, Moscow 125412, Russia

The model is developed of near-cathode layers in high-pressure arc discharges. Governing equations include equations of conservation of the ions, the atoms, and the electrons, transport equations for the ions, the atoms, and the electrons, equations of energy for the electrons and the heavy particles, and the Poisson equation. The equations are solved numerically in 1D without any further simplifications, in particular, without explicitly dividing the near-cathode layer into a space-charge sheath and quasi-neutral plasma. Results of numerical simulation of the layer in atmospheric-pressure argon arc with tungsten cathode are given: distributions of plasma parameters across the layer, current-voltage characteristics, energy flux from the plasma to the cathode. The results are compared with those obtained in the framework of an approximate approach.

1. Introduction

A correct description of near-cathode plasma layers in high-pressure arc discharges is of key importance in the theory of arc plasma-cathode interaction. These layers have a complex structure, consisting of two main sublayers: space-charge sheath and ionization layer. Existing models of the near-cathode layers are based on separate consideration of the two sheaths [1-4]. Such approach requires the use of an approximate procedure of matching solutions at the sheath boundary. Moreover, existing models use some simplifying assumptions (e.g., the constancy of species temperatures inside the ionization layer). In this work, current transfer through near-cathode layers is considered on the basis of direct numerical simulation of the layer, without dividing it into the sheaths and without using assumptions concerning spatial distributions of plasma parameters.

2. The model

The plasma layer near a hemispherical cathode is considered in one-dimensional (1D) approximation, with spherical symmetry (under the assumption that the electric current is distributed uniformly over the spherical part of the cathode surface). The use of a 1D approach is justified, because in typical conditions the width of the layer is much smaller than the cathode radius.

The system of equations governing spatial distributions of plasma parameters is as follows. Equations of conservation of species are written as

$$\nabla \cdot \mathbf{J}_\alpha = \omega_\alpha, \qquad \alpha = i, e, a. \tag{1}$$

Here n_α, v_α, and $\mathbf{J}_\alpha = n_\alpha \mathbf{v}_\alpha$ are the number densities, velocities, and number densities of transport fluxes of plasma components, indexes i, e, a refer to ions, electrons, and atoms, respectively, ω_α are the net rates of production of particles of the type α in volume reactions,

$$\omega_i = \omega_e = -\omega_a = k_i n_a n_e - k_r n_i n_e^2, \tag{2}$$

where k_i and k_r are the ionization and recombination rate constants.

Adding Eqs. (1) for the ions and the atoms, one arrives at the equation of conservation of the gas nuclei

$$\nabla \cdot (\mathbf{J}_i + \mathbf{J}_a) = 0. \tag{3}$$

Subtracting Eq. (1) for the electrons from that for the ions, one arrives at the equation of continuity of the electric current

$$\nabla \cdot (\mathbf{J}_i - \mathbf{J}_e) = 0. \tag{4}$$

Transport equations taking into account the multicomponent diffusion, resolved with respect to the diffusion forces (Stefan-Maxwell equations), may be written as [5]

$$-\nabla p_\alpha + n_\alpha e Z_\alpha \mathbf{E} + \frac{\rho_\alpha}{\rho}[\nabla p - e(n_i - n_e)\mathbf{E}] - \sum_\beta \frac{n_\alpha n_\beta k T_{\alpha\beta} C_{\alpha\beta}}{n D_{\alpha\beta}}(\mathbf{v}_\alpha - \mathbf{v}_\beta) - \mathbf{R}_\alpha^T = 0, \tag{5}$$

where

$$T_{\alpha\beta} = \frac{m_\alpha T_\beta + m_\beta T_\alpha}{m_\alpha + m_\beta}, \qquad D_{\alpha\beta} = \frac{3\pi}{16}\left(\frac{2kT_{\alpha\beta}}{\pi m_{\alpha\beta}}\right)^{1/2} \frac{1}{n\overline{Q}_{\alpha\beta}^{(1,1)}}. \tag{6}$$

Here $\alpha, \beta = i, e, a$, T_α and $p_\alpha = n_\alpha k T_\alpha$ are the temperatures and partial pressures of plasma components (the temperatures of atoms and ions are assumed equal), $n=\sum_\beta n_\beta$ and $p=\sum_\beta p_\beta$ are the total number density and pressure, $D_{\alpha\beta}$ are the binary diffusion coefficients evaluated in the first approximation of the Chapman-Enskog method, $m_{\alpha\beta}$ are the reduced masses, $\overline{Q}_{\alpha\beta}^{(1,1)}$ are the energy-averaged transport cross sections, coefficients $C_{\alpha\beta}$ introduce corrections due to account of approximations of higher orders (note that $C_{\alpha\beta}=C_{\beta\alpha}$). The thermal diffusion forces $\mathbf{R}_\alpha^T$ are given by formulas

$$\mathbf{R}_\alpha^T=C_\alpha^{(h)}n_\alpha k\nabla T_h+C_\alpha^{(e)}n_\alpha k\nabla T_e, \mathbf{R}_e^T=C_e^{(e)}n_e k\nabla T_e,$$

for heavy-particle species ($\alpha\neq e$) and for electrons, respectively. (Note that the transport equation for electrons does not contain a term with ∇T_h since the corresponding force is negligibly small due to the smallness of the electron-to-ion mass ratio [5]). The thermal diffusion coefficients $C_\alpha^{(h)}$ and $C_\alpha^{(e)}$ satisfy the following equalities:

$$\sum_{\alpha\neq e} n_\alpha C_\alpha^{(h)}=0,\quad \sum_\alpha n_\alpha C_\alpha^{(e)}=0. \qquad (7)$$

Eqs. (5) are dependent, therefore any one of them may be dropped. We will drop the equation for atoms.

The assumption of no convection requires that the force applied by the electric field be compensated by the plasma pressure gradient

$$-\nabla p+e(n_i-n_e)\mathbf{E}=0. \qquad (8)$$

We introduce the x-axis which is directed from the cathode surface into the plasma and designate by J_α and E the x-components of the vectors $\mathbf{J}_\alpha$ and $\mathbf{E}$. Eqs. (3) and (4) may be integrated to give

$$J_i+J_a=0, \qquad (9)$$

$$J_i-J_e=-\frac{j}{e}. \qquad (10)$$

Eq. (9) was written with account of the fact that the nuclei do not accumulate nor disappear at the cathode surface, j in Eq. (10) is the density of electric current directed from the plasma to the cathode.

The electron and heavy-particle energy equations can be written as

$$\frac{5}{2}\nabla\cdot(p_e\mathbf{v}_e)=-\nabla\cdot\mathbf{h}_e-en_e\mathbf{v}_e\cdot\mathbf{E}-\frac{3n_e k^2T_e}{m_i n}(T_e-T_h)\left(\frac{n_a}{D_{ea}}+\frac{n_i}{D_{ei}}\right)-w_e^{(e)}, \qquad (11)$$

$$-\nabla\cdot\mathbf{h}_{hp}+en_i\mathbf{v}_i\cdot\mathbf{E}+\frac{3n_e k^2T_e}{m_i n}(T_e-T_h)\left(\frac{n_a}{D_{ea}}+\frac{n_i}{D_{ei}}\right)=0 \qquad (12)$$

(note that the term $\frac{5}{2}\nabla\cdot(p_a\mathbf{v}_a+p_i\mathbf{v}_i)$ equals to zero, that follows from Eq. (9) and the equality of ion and atom temperatures). Here $\mathbf{h}_e$ and $\mathbf{h}_{hp}$ are the densities of the electron and heavy-particle heat fluxes; the term $w_e^{(e)}$ in Eq. (11), describing energy losses due to inelastic collisions, may be evaluated as [6]

$$w_e^{(e)}=E_i\omega_e+w_{rad}, \qquad (13)$$

where E_i is the ionization potential for atoms and w_{rad} represents radiation energy losses.

Expressions for kinetic and transport coefficients used in our calculations are taken from [6] with the use of electron-atom and atom-atom cross sections for argon from [7] and [8], respectively. The ionization rate is calculated according to [9]; the radiation losses are evaluated using the data presented in [10].

The system includes also the Poisson equation

$$\varepsilon_0\nabla\cdot\mathbf{E}=e(n_i-n_e). \qquad (14)$$

A numerical solution of the above system of equations is sought in the region $r_c<x<L$, where the lower boundary corresponds to the position of the cathode surface and the upper boundary is placed far from the cathode, in the region of thermal and ionization equilibrium. The boundary conditions for the above equations are as follows. Assuming that the cathode surface absorbs all ions coming from the plasma, one can set the ion density at this surface equal to zero (see the discussion in [9, Appendix B])

$$n_i(r_c)=0. \qquad (15)$$

The boundary condition for the electron density at the cathode, that accounts the emission current and the electron current backscattered to the cathode due to thermal motion, may be written as

$$x=r_c:\ \frac{j_{em}}{e}-\frac{n_e C_e}{4}=J_e \qquad (16)$$

where $C_e=(8kT_e/\pi m_e)^{1/2}$ is the thermal velocity of electrons. The emission current density j_{em} is determined by means of the Richardson-Schottky equation.

The boundary conditions on the plasma side of the region considered correspond to the ionization equilibrium

$$n_i(L)=n_e(L)=n_{eq}. \qquad (17)$$

It is necessary to specify also a difference U between potential of the plasma at $x = L$ and at the cathode. This difference is positive and can be viewed as a near-cathode voltage drop.

The boundary conditions for Eqs. (11) and (12) at the cathode are

$$x = r_c : \begin{cases} \dfrac{j_{em}}{e} 2kT_w - \dfrac{n_e C_e}{4} 2kT_e = \dfrac{5}{2} p_e v_e + h_e \\ T_h = T_w \end{cases} \quad (18)$$

where h_e is the x-component of the heat flux $\mathbf{h}_e$, T_w is the cathode surface temperature (a given parameter).

At the plasma side, $T_e(L)$ is given (and equal to $T_h(L)$).

Summing Eqs. (11) and (12) with account of Eq. (13), one obtains

$$\nabla \cdot \left[\left(\frac{5}{2} kT_e + E_i \right) \mathbf{J}_e + \mathbf{h}_e + \mathbf{h}_{hp} \right] = \mathbf{E} \cdot \mathbf{j} - w_{rad} . \quad (19)$$

Addition to the left-hand side of Eq. (19) of the term $\nabla \cdot [(A - E_i)\mathbf{j}/e]$ (here A is the work function for the cathode material), equal to zero according to Eq. (10), allows one to rewrite Eq. (19) in the form

$$\nabla \cdot \left[\left(\frac{5}{2} kT_e + A \right) \mathbf{J}_e + (E_i - A) \mathbf{J}_i + \mathbf{h}_e + \mathbf{h}_{hp} \right] = \mathbf{E} \cdot \mathbf{j} - w_{rad} . \quad (20)$$

The density q_c of the energy flux to the cathode surface is

$$q_c = - \left[\left(\frac{5}{2} kT_e + A \right) J_e + (E_i - A) J_i + h_e + h_{hp} \right]_{x=r_c} .$$

3. Results and discussion

Calculations of the near-cathode layer in atmospheric-pressure argon have been performed for the hemispherical tungsten cathode of radius $r_c = 1$ mm. In figure 1 an example is given of distributions of plasma parameters (electron and ion number densities, electron and heavy-species temperatures, equilibrium (Saha) electron number density, electric field and potential difference between the cathode surface and the point being considered) at $T_w = 3000$ K, for the total current $I = 10$ A, corresponding to the current density at the cathode $j_c = I/(2\pi r_c^2) = 1.6 \times 10^6$ A/m². It is seen that the space-charge sheath, where the major increase of potential occurs, has a width about 3 µm. The densities reach the equilibrium density at approximately 300 µm, also at this distance the electron and the heavy particle temperatures are approximately the same.

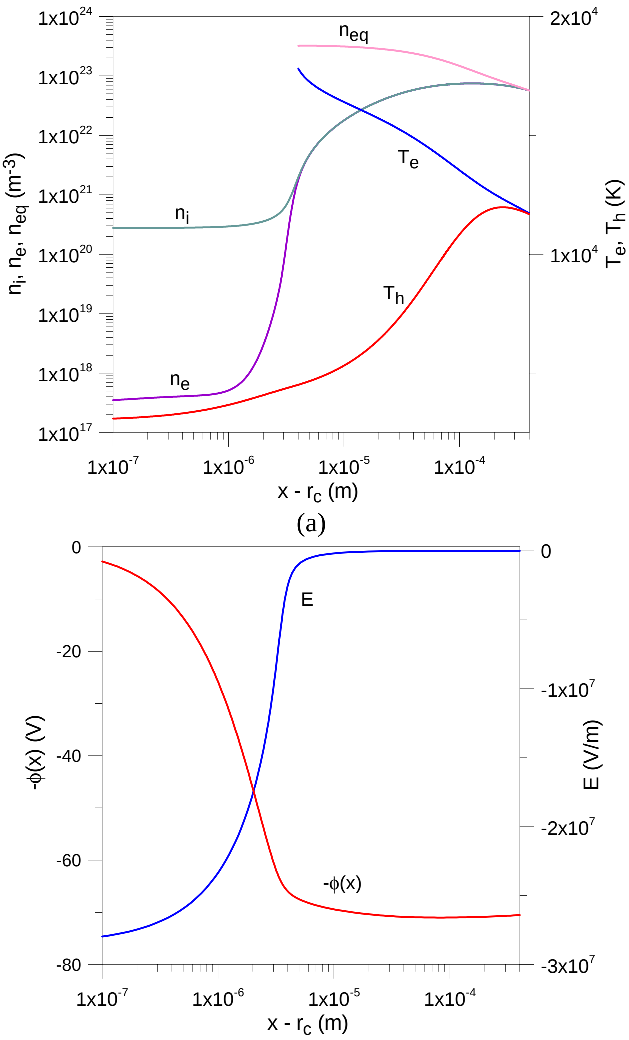

Figure 1. Distributions, along the axis, of the electron and ion densities, equilibrium electron number density and electron and heavy-particle temperatures (a) and the electric field and potential difference (b) at the cathode temperature 3000 K and electric current 10 A.

In figure 2 the current-voltage characteristic is given for $T_w = 3000$ K. The total current density and its components (the electron and ion current densities) at the cathode are also given. The figure presents also the density of the energy flux to the cathode and the mean electron temperature in the ionization layer versus the potential of the plasma. The character of the presented dependences is similar to that obtained using a simplified approach [2,4].

In figure 3 the electric current and the density of the energy flux to the cathode are given, versus the cathode temperature, for potentials of the plasma 12

V and 25 V. Results obtained using the model [4] are also given. It is seen that the results of direct calculations are rather close to those obtained in the framework of the approach based on separate consideration of the space-charge and ionization sheaths.

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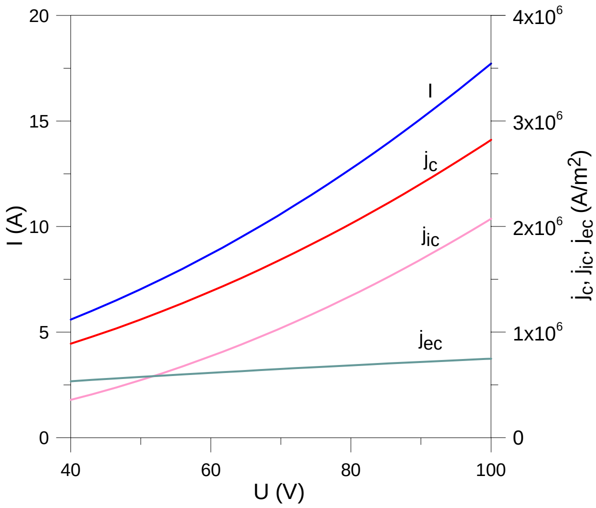
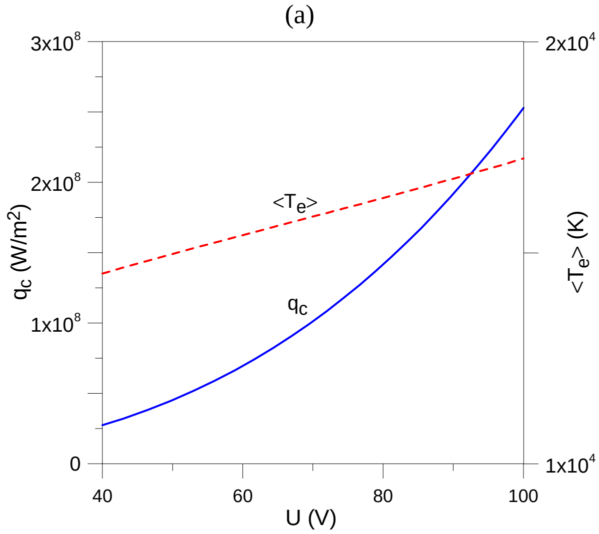


(a)



(b)

Figure 2. The electric current, the total current density and the electron and ion current densities at the cathode (a) and the density of the energy flux to the cathode and the mean electron temperature in the ionization layer (b) versus the potential of the plasma at the cathode temperature 3000 K.

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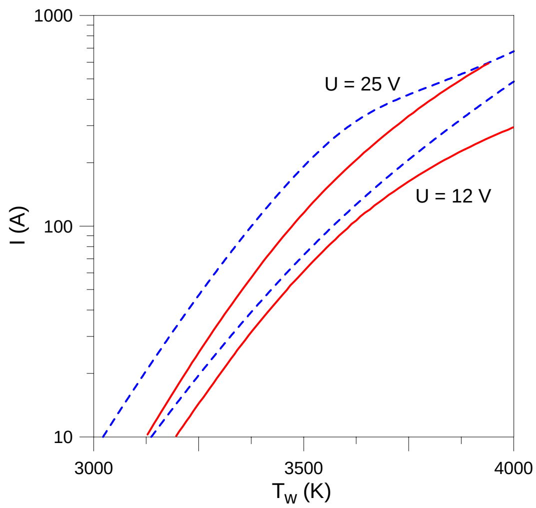
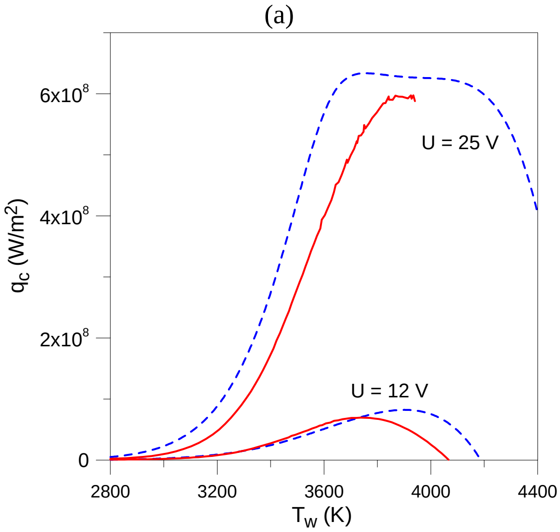


(a)



(b)

Figure 3. The electric current (a) and the density of the energy flux to the cathode (b) versus the cathode temperature for potentials of the plasma 12 V and 25 V. Solid lines - this model, dashed lines - the model [4].