Understanding bifurcations encountered in numerical modelling of current transfer to cathodes of DC glow and arc discharges

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This work is concerned with analysis of bifurcations that have been encountered in the modelling of steady-state current transfer to cathodes of glow and arc discharges. All basic types of steady-state bifurcations (fold, transcritical, pitchfork) have been identified and analyzed. The analysis provides explanations to many results obtained in numerical modelling. In particular, it is shown that dramatic changes of patterns of current transfer to cathodes of both glow and arc discharges, described by numerical modelling, occur through perturbed transcritical bifurcations of first and second order contact. The analysis elucidates the reason why the mode of glow discharge associated with the falling section of the CVC in the solution of von Engel and Steenbeck seems not to appear in 2D numerical modelling and the subnormal and normal modes appear instead. A similar effect has been identified in numerical modelling of arc cathodes and explained.

1. Motivation

Bifurcations of current transfer to cathodes of DC gas discharges or their consequences are sometimes encountered in apparently simple situations and a failure to recognize and properly analyze a bifurcation may originate difficulties in numerical modelling and hinder understanding of numerical results and the underlying physics. As an example, figure 1 shows the current-voltage characteristics (CVCs) of a DC glow discharge calculated in the framework of a simple drift-diffusion model under different approximations: in one dimension (1D) without account of diffusion of the ions and the electrons; in 1D with account of (axial) diffusion; in two dimensions (2D) under the approximation of axial symmetry with account of diffusion both in the axial direction and to the (absorbing) wall.

The 1D solutions with and without diffusion are rather close to each other and represent in essence the classic solution of von Engel and Steenbeck (e.g., [1]). The 2D solution is close to the 1D solution with account of diffusion at low and high currents, however at intermediate currents it describes the subnormal and normal modes rather than the mode associated with the falling section of the CVC. Note that the ratio of the electron current to the wall of the discharge tube to the discharge current, evaluated with the use of the 2D solution, is of the order of 10^{-3} or lower at all discharge currents. In other words, diffusion of the charged particles to the wall is a weak effect and a question arises how this weak effect originates such a large difference, in particular, where from have the subnormal and normal modes appeared and where to has the mode associated with the falling section of the CVC gone. These questions will be addressed in the present work, and the identification and understanding of the relevant bifurcation are indispensable here.

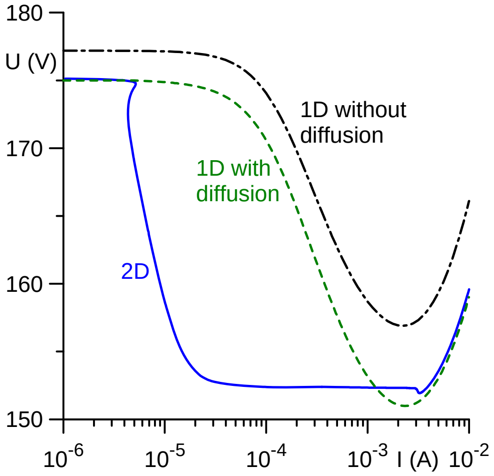

Figure 1. CVCs of the glow discharge. Xe plasma, p = 30 Torr, the discharge radius 1.5 mm and height 0.5 mm.

As another example, figure 2 shows the CVC of the near-cathode region and temperatures T_c and T_e at the center and, respectively, edge of the front surface of a cylindrical arc cathode, calculated by means of the Internet tool [2]. The code starts from a 1D initial approximation, describing the diffuse mode on a cathode with an insulated lateral surface, and then gradually eliminates the insulation until a solution for a fully active lateral surface has been found. Under the conditions considered, this approach works nicely at the near-cathode voltages U below approximately 13.46 V. The obtained CVC $U(I)$ is falling and $T_c < T_e$; typical features of the diffuse mode of operation of an arc cathode. There is no convergence at U between 13.46 V and 14.04 V. The convergence re-appears at $U \gtrsim 14.04$ V. The CVC remains falling, however $T_c > T_e$: it looks like

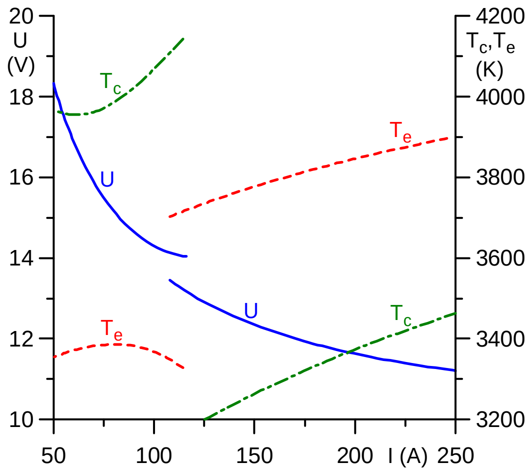

Figure 2. CVCs of the near-cathode region of the arc discharge and temperatures at the center and the edge of the front surface of the cathode. Simulation by means of the Internet tool [2] with the use of the built-in initial approximation. Ar plasma, p = 1 bar, W cathode of 2 mm radius and 10 mm height.

a mode with a spot at the center of the front surface of the cathode. Questions arise why simulations which start from the diffuse mode on a cathode with the insulating lateral surface are unable to arrive at the diffuse mode on a cathode with the active lateral surface and what significance has the value U = 13.46 V at which the troubles start. These questions also will be addressed in the present work and will be shown to be related to the preceding ones.

2. Models and numerics

The calculation domain is the interelectrode gap in the case of glow discharge and the body of a thermionic cathode in the case of arc discharge, and is assumed to be a circular cylinder of a radius R and a height h (except in the case of an arc cathode that is treated when the transcritical bifurcation of second order contact is considered, and where it has a hemispherical tip).

The distribution of the ion and electron densities n_i and n_e and the electrostatic potential Φ in the interelectrode gap in the case of a glow discharge is described by the simplest self-consistent model which comprises equations of conservation of a single ion species and the electrons written in the drift-diffusion approximation (e.g., [3]) with account of electron impact ionization and dissociative recombination, and the Poisson equation. Results reported in this work refer to a discharge in xenon under the pressure of 30 Torr, R = 1.5 mm, and h = 0.5 mm.

The simulation of the interaction of high-pressure arc plasmas with thermionic cathodes is based on the model of nonlinear surface heating; e.g., [4]. In the framework of this model, the distribution of the temperature T inside the cathode is found by solving the equation of heat conduction with a nonlinear boundary condition that describes the energy exchange of the cathode with the adjacent plasma. Densities of the energy flux and the electric current to the cathode surface are treated as known functions of the local temperature and the near-cathode voltage drop U: $q = q(T,U)$ and $j = j(T,U)$. These functions are calculated in advance by means of solving equations describing the near-cathode plasma layer in a high-pressure plasma which are summarized in [5]. Results reported in this work refer to an arc in atmospheric-pressure argon with a tungsten cathode, R = 2 mm, and h = 10 mm.

Let us introduce cylindrical coordinates (r,φ,z) with the axis z coinciding with the axis of the calculation domain and with the origin at the center of the surface of the glow cathode or at the center of the front surface of the arc cathode. In the case of glow discharge, the boundary conditions at $z = 0$ and $z = h$ are the conventional ones: at $z = 0$, i.e., at the cathode, the ion diffusion current is neglected, the electron current is due to secondary emission, and the electrostatic potential vanishes; at $z = h$, i.e., at the anode, the electron diffusion current is neglected, the ion density vanishes, and the electrostatic potential equals the discharge voltage U. In the case of arc cathode, the front surface of the cathode, $z = 0$, is heated by the adjacent plasma and the boundary condition is $\kappa(\partial T/\partial z) = -q(T,U)$. The temperature at $z = h$, i.e., at the cathode base, is governed by the cooling arrangement and was set equal to 293 K.

In the case of glow discharge, one boundary condition at $r = R$, i.e., at the wall of the discharge tube, is zero density of electric current. The boundary conditions at $r = R$ for n_i and n_e are written in the form

$$s n_{i,e} + (1-s)\frac{\partial n_{i,e}}{\partial r} = 0, \qquad (1)$$

where s is a given parameter that varies between 0 and 1. $s = 0$ corresponds to a (totally) reflecting wall, or, in other words, to losses of the charged particles due to their diffusion to the wall being neglected. $s = 1$ corresponds to an absorbing wall, or, in other words, to diffusion losses being taken into account. In the case of arc cathode, the boundary condition at $r = R$, i.e., at the lateral surface of the cathode, is written as

$$\kappa\frac{\partial T}{\partial r} = s q(T,U), \qquad (2)$$

where s again is a given parameter varying between 0 and 1. $s = 0$ corresponds to the lateral surface of the cathode being thermally (and electrically) insulated, $s = 1$ corresponds to the lateral surface being active, i.e., energy- and current-collecting.

The above-stated problems admit axially symmetric (2D) solutions, $f = f(r,z)$, 3D solutions, $f = f(r,\varphi,z)$, and, in the particular case $s = 0$, also 1D solutions, $f = f(r)$. Here f designates the set of quantities n_i, n_e, Φ in the case of glow discharge and T in the case of arc cathode. In this work, 1D and 2D steady-state solutions for the glow discharge and 1D, 2D, and 3D solutions for the arc cathode are considered. Solutions for the glow discharge and 3D solutions for the arc cathode were calculated with the use of the commercial finite element software COMSOL Multiphysics. 1D and 2D solutions for the arc discharge were calculated with the use of the tool [2], except in the case of an arc cathode with a hemispherical tip that is treated when transcritical bifurcation of second order contact is considered, where COMSOL Multiphysics was employed.

Data on points of transcritical and pitchfork bifurcations of steady-state solutions reported in this work were obtained as follows. In the case of glow discharge, one of the bifurcating solutions is 1D and the other is 2D [these bifurcations will be designated {1D,2D} from now on] and the bifurcation points were found by means of solving the appropriate eigenvalue problem with the use of COMSOL Multiphysics. In the case of arc cathode, bifurcations {1D,2D}, {1D,3D}, {2D,2D}, {2D,3D}, and {3D,3D} are present. Points of bifurcations {1D,2D}, {1D,3D}, and {2D,3D} were calculated with the use of the tool [2], points of bifurcations {2D,2D} and {3D,3D} were calculated with the use of COMSOL Multiphysics.

Data on stability of current transfer to arc cathodes reported in this work were obtained by means of solving the appropriate eigenvalue problem with the use of COMSOL Multiphysics as described in [6].

3. Results

All basic types of steady-state bifurcations (fold, transcritical, pitchfork) are encountered in numerical modelling of current transfer to cathodes of DC glow and arc discharges. However, care is needed in the choice of the coordinates of the bifurcation diagram in order to easily identify the bifurcations. For example, the CVC represents a proper bifurcation diagram only for fold bifurcations.

A fold bifurcation represents not branching of essentially different modes but rather a turning point of the same mode. The importance of understanding these bifurcations originates in their relation with stability of current-controlled discharges, that can be formulate as follows: a steady-state mode which is stable against perturbations of the fundamental mode can turn only clockwise, after which it becomes unstable; an unstable mode can turn only counterclockwise, after which it becomes stable.

It happens frequently that a steady-state mode manifests more than one turning point. According to the above, there is a change of stability at each point against a mode of perturbations. It seems legitimate to assume that the perturbation mode against which the change of stability occurs is the same for all turning points of the steady-state mode in question. (In the case of arc cathode, this assumption was confirmed by the numerical modelling.) According to the above rule, a steady-state mode that manifests more than one turning point can not manifest 360°-loops.

Transcritical bifurcations encountered are of two types: transcritical bifurcations of first order contact, where two steady-state modes intersect at the bifurcation point, and transcritical bifurcations of second order contact, where two steady-state modes are tangent at the bifurcation point.

In the case of glow discharge with $s = 0$, the 2D spot modes join the diffuse mode at two transcritical bifurcation points of first order contact. One of these points is positioned in the vicinity of the point of minimum of the CVC of the diffuse mode, and the other, at low currents. In the case of arc cathode with $s = 0$, the 2D spot mode joins the diffuse mode through one transcritical bifurcation points of first order contact. This point is positioned in the vicinity of the point of minimum of the CVC of the diffuse mode.

Results of an investigation of stability of the diffuse and 2D spot modes of current transfer to an arc cathode have revealed an exchange of stability in the vicinity of points of transcritical bifurcation of first order contact. It is natural to assume that in the case of glow discharge the conclusions on stability apply to the transcritical bifurcation of first order contact.

The transcritical bifurcations of first order contact are perturbed when $s \neq 0$: the steady-state modes that exist at $s = 0$ are broken and two scenarios can occur. Perturbed transcritical bifurcations of first order contact produce dramatic changes of patterns of DC current transfer in both glow and arc discharges.

Numerical modelling of glow discharge and arc cathode revealed transcritical bifurcations of second order for a particular value R_{ts} of the discharge tube radius in the case of the glow discharge with reflecting wall or the cathode with a hemispherical tip in the case of the arc cathode with a lateral surface active.

The transcritical bifurcations of second order contact are perturbed when $R \neq R_{ts}$. In the case of the glow discharge, this causes 2D spot modes to disappear one by one, starting from the higher-order modes. In the case of arc cathodes, the perturbation of the bifurcation causes the appearance of two new disconnected modes that embraces states typical for both diffuse and spot modes and cannot be termed (pseudo)diffuse or spot mode. Hence, perturbed transcritical bifurcations of second order contact also produces dramatic changes of patterns of DC current transfer in both glow and arc discharges.

Pitchfork bifurcations represent branching of two or more modes with a lower degree of symmetry than that of the mode from which they branch off. The simulations have been performed only for arc cathodes and breaking of axial or planar symmetries was observed. A 3D spot mode that branches off from another 3D spot mode as a result of breaking of planar symmetry is found for the first time. It is natural to assume that the conclusions on pitchfork bifurcations {1D,3D} and {2D,3D}, drawn for the case of arc cathode, apply also to the case of glow discharge. A bifurcation {3D,3D} similar to the one discussed above for the case of arc cathode can occur also in the case of glow discharge.

4. Conclusions

The numerical modelling of current transfer to cathodes of DC glow and arc discharges exhibited all basic types of steady-state bifurcations. However, dramatic changes of patterns of DC current transfer occur in both glow and arc discharges only through perturbed transcritical bifurcations of first and second order contact.

Analysis of bifurcations allows one to understand main features of patterns of steady-state modes and their stability. For example, the analysis elucidates the reason why the mode associated with the falling section of the CVC in the classic 1D solution of von Engel and Steenbeck seems not to appear in 2D numerical modelling and the subnormal and normal modes appear instead. A similar effect has been identified in numerical modelling of arc cathodes and explained.

Multiple modes of current transfer to DC discharge cathodes represent a self-organization phenomenon. In spite of physical mechanisms of discharges on cold and hot cathodes being very different, the self-organization fits into the same pattern. However, there are some differences, for example different scenarios of exchange of branches in breaking of transcritical bifurcations of first order contact. This difference originates in the fact that an absorbing wall locally quenches the glow discharge due to loss of the charged particles caused by diffusion to the wall, while lateral heating of an arc cathode increases the temperature of the cathode edge and thus locally enhances the discharge. Another difference is that the fundamental mode of a glow discharge in a tube with an absorbing wall manifests a normal spot, while the fundamental mode on an arc cathode with a current- and energy-collecting lateral surface is characterized by modest variations of parameters along the front surface of the cathode and is in this respect similar to the diffuse mode. This difference stems from the essentially different aspect ratio. A further difference is that modes with regular patterns of two or more spots are not observed on arc cathodes, but have been observed in glow discharges; see e.g. [7]. This difference remains to be explained.

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