Modelling self-organization on DC glow cathodes

P. G. C. Almeida, M. S. Benilov, and M. J. Faria

Departamento de Física, Universidade da Madeira, Largo do Municipio, 9000 Funchal, Portugal

Multiple steady-state solutions existing in the theory of glow discharges are computed. The simulations are 2D and performed in the framework of the simplest self-consistent model of glow discharge, which accounts for a single ion species and employs the drift-diffusion approximation. Solutions describing up to nine different modes were found. One mode is 1D and represents in essence the well-known solution of von Engel and Steenbeck. The other eight modes are axially symmetric, exist in limited ranges of the discharge current, and are associated with different patterns of current spots on the cathode. Account of diffusion losses affects the solutions strongly. The solutions that exist in limited current ranges describe patterns which may be viewed as axially symmetric analogues of the 3D patterns observed in DC glow microdischarges in xenon.

1. Introduction

It is well known that current transfer to cathodes of DC glow discharges can occur in the abnormal mode or in the mode with a normal spot. Recently, also modes with multiple spots have been observed; e.g., [1,2] and references therein. These facts, jointly with considerations stemming in bifurcation analysis and the theory of nonlinear dissipative structures, have led to a hypothesis [3-5] that a self-consistent theoretical model of a near-cathode region in a DC glow discharge must admit multiple steady-state solutions describing different modes of current transfer. However, these solutions have not been revealed by the numerical modelling [6-12].

For the first time, these solutions have been reported in [13]. In [13], diffusion losses were not accounted for and only two 2D modes were found. In the present work, additional data on the multiple solutions without diffusion losses are given and the effect of the diffusion losses is investigated.

2. The model

Let us consider a mathematical model of a DC glow discharge comprising equations of conservation of a single ion species and the electrons, transport equations for the ions and the electrons written in the so-called drift-diffusion approximation, and the Poisson equation. The processes considered for charged particle production and decay are electron impact ionization and dissociative recombination. The temperatures of the charged particles are assumed given and uniform throughout the discharge.

Boundary conditions at the cathode and anode are written in the conventional form: diffusion fluxes of the attracted particles are neglected as compared to drift; electrons are emitted by the cathode through secondary emission; density of ions vanishes at the anode; electrostatic potentials of both electrodes are given. One boundary condition at the wall of the discharge vessel is the conventional condition of zero electric current density. Two boundary conditions are used alternatively for the densities of charged particles at the wall, corresponding to the cases i) where diffusion losses to the wall are neglected, and ii) where diffusion losses are taken into account.

Results reported in this work refer to a discharge in xenon under the pressure of 30 Torr. The interelectrode gap is h = 0.5 mm and the radius of the discharge tube R is between 1.5 mm or 0.5 mm . The mobility of Xe_2^+ ions in Xe was evaluated by means of the formula $\mu_i = 2.1\times10^{21}\ \mathrm{m^{-1}\ V^{-1}\ s^{-1}} / n_n$ (here n_n is the density of the neutral gas). The mobility of the electrons was evaluated by the formula

$\mu_e = 17\ \mathrm{Torr\ m^{-2}\ V^{-1}\ s^{-1}} / p$, where p is the pressure of the plasma; Townsend's ionization coefficient α was evaluated by equation (4.6) of [14]. The diffusion coefficients were evaluated by means of Einstein's law and the temperatures of the heavy particles and electrons were set equal to, respectively, T_i = 300 K and T_e = 1 eV. The coefficient of dissociative recombination of molecular ions Xe_2^+ was set equal to $2\times10^{-13}\ \mathrm{m^3\ s^{-1}}$. The effective secondary emission coefficient was set equal to 0.03. Cylindrical coordinates (r, φ, z) with the origin at the center of the cathode and the z-axis coinciding with the axis of the vessel were employed.

Numerical results reported in this work have been calculated with the use of the commercial finite element software COMSOL Multiphysics. The 1D mode, which exists at all discharge currents and is termed fundamental, may be found as a matter of routine. However, other solutions are not easy to find: one needs to know in advance what these solutions are like and where they should be sought. In this work, this information was obtained by means of finding bifurcation points where 2D solutions branch off from the fundamental mode, similarly to what has been done in [5].

3. Multiple solutions without diffusion losses

Results reported in this section refer to a discharge tube of the radius R = 1.5 mm. Multiple solutions describing nine different modes were detected in this case, a 1D mode and eight 2D modes. As an example, the current-voltage characteristics (CVCs) of the fundamental mode and the first, fifth and eighth 2D modes are shown in figure 1. Here $<j>$ is the average value of the axial component of the electric current density evaluated over the (circular) cross section of the discharge vessel. It should be noted that the CVC of the eighth 2D spot mode coincides, to the graphical accuracy, with the CVC of the fundamental mode. Each of the 2D modes exists in a limited range of the discharge current and its CVC represents a closed curve.

For each of the 2D modes, there are two states in which densities of charged particles and electric potential vary with z but not with r. These states are marked by circles in figure 1 and subsequent figures. One of these two states is close to the point of minimum of the CVC of the fundamental mode and is designated a_i (i = 1,2,..,8), the other is

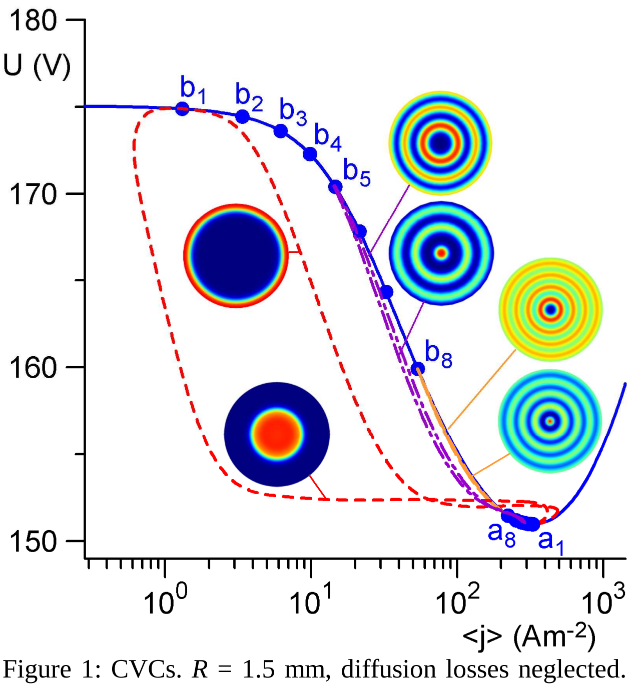

Figure 1: CVCs. R = 1.5 mm, diffusion losses neglected. Solid: fundamental mode; dashed: first 2D mode; dashed-dotted: fifth 2D mode; circles: bifurcation points.

designated b_i and located at lower currents. These states belong not only to the 2D mode being considered, but also to the fundamental mode. In other words, the solutions describing the 2D mode and the fundamental mode coincide at these states. This phenomenon is well known in mathematical physics and called bifurcation, or branching, of solutions, and states where it occurs are called bifurcation points.

The (two) bifurcation points positioned on each 2D mode divide this mode into two branches. Schematics in figure 1 illustrate current density distributions over the cathode surface corresponding to each branch. One of the two branches of each 2D mode is associated with a pattern comprising a spot at the centre of the cathode and the other with a pattern without a central spot.

The complexity of the patterns associated with the 2D modes increases with i or, equivalently, with a decrease of the range of existence of the corresponding mode.

For the central-spot branch of the first 2D mode a plateau on the CVC can be seen in figure 1. The maximum of current density over the cathode surface for states belonging to this plateau does not change much. Both the plateau of the CVC and the approximate constancy of current density inside the spot are characteristic features of the effect of normal current density. Hence, the section of the central-spot branch of the first 2D mode corresponding to the plateau of the CVC may be identified with the normal discharge.

The effect of normal current density is manifested also by the other branch of the first 2D mode and by the second and subsequent 2D modes, however with increasing i it becomes less pronounced and occurs in a narrower range of the discharge currents, eventually disappearing.

The patterns with multiple spots may be viewed as axially symmetric analogues of 3D patterns observed in DC glow microdischarges, see e.g. [1,2]. Thus, the present modelling supports the hypothesis [3-5] that patterns with multiple spots may be described in the framework of basic mechanisms of glow discharge, so there is no need to introduce special mechanisms to this end.

4. The effect of variation of the radius

Calculations for a discharge tube of the radius R = 0.5 mm revealed only two 2D modes; see figure 2. Also shown in this figure is the fundamental mode (which is the same that the one found for R = 1.5 mm). Average current densities corresponding to the bifurcation points positioned on the first and second 2D modes are $\langle j(a_1)\rangle = 311$ A m^{-2}, $\langle j(b_1)\rangle = 7.7$ A m^{-2}, $\langle j(a_2)\rangle = 265$ A m^{-2}, and $\langle j(b_2)\rangle = 26$ A m^{-2}. Note that the similar values for R = 1.5 mm are $\langle j(a_1)\rangle = 329$ A m^{-2}, $\langle j(b_1)\rangle = 1.3$ A m^{-2}, $\langle j(a_2)\rangle = 325$ A m^{-2}, and $\langle j(b_2)\rangle = 3.4$ A m^{-2}. One can conclude that a decrease of R causes a shift of the two bifurcation points belonging to each 2D mode in the directions towards each other. The range of existence of each of the 2D modes also shrinks with a decrease of R.

Calculations for intermediate R have shown that the third to eighth 2D modes disappear with decreasing R one by one, and the disappearance occurs through shrinking of their existence ranges. A more or less pronounced effect of normal current density is present at R = 0.5 mm only on the first 2D mode, as evidenced by the CVCs shown in figure 2.

5. Solutions with diffusion losses

When diffusion losses to the wall are taken into account, the fundamental mode is no longer 1D but rather becomes 2D. Furthermore, the non-fundamental 2D modes do not bifurcate from the fundamental mode. Under these circumstances it is not possible to rely on bifurcation analysis in order to obtain information on the range of existence of non-fundamental 2D modes or what these modes are like.

The procedure of finding these modes was as follows. Starting from a state belonging to a 2D mode without diffusion losses, the diffusion losses are gradually introduced for a fixed value of the input parameter (the discharge voltage or current). When diffusion losses have been fully introduced, the next step was to vary the input parameter in order to obtain the 2D mode in the whole range of its existence.

Three 2D modes have been found for R = 1.5 mm. One of them exists at all current values, i.e., is fundamental, the other two modes exist only in limited current ranges. The CVCs of the three modes are shown in figure 3. The 2D modes are all disconnected from each other, although CVCs intersect at some values of the discharge current. The CVCs of the non-fundamental 2D modes represent closed curves and each is constituted by two branches separated by two turning points. Also shown in figure 3 is the fundamental mode for R = 0.5 mm. No other 2D modes have been found for this value of R.

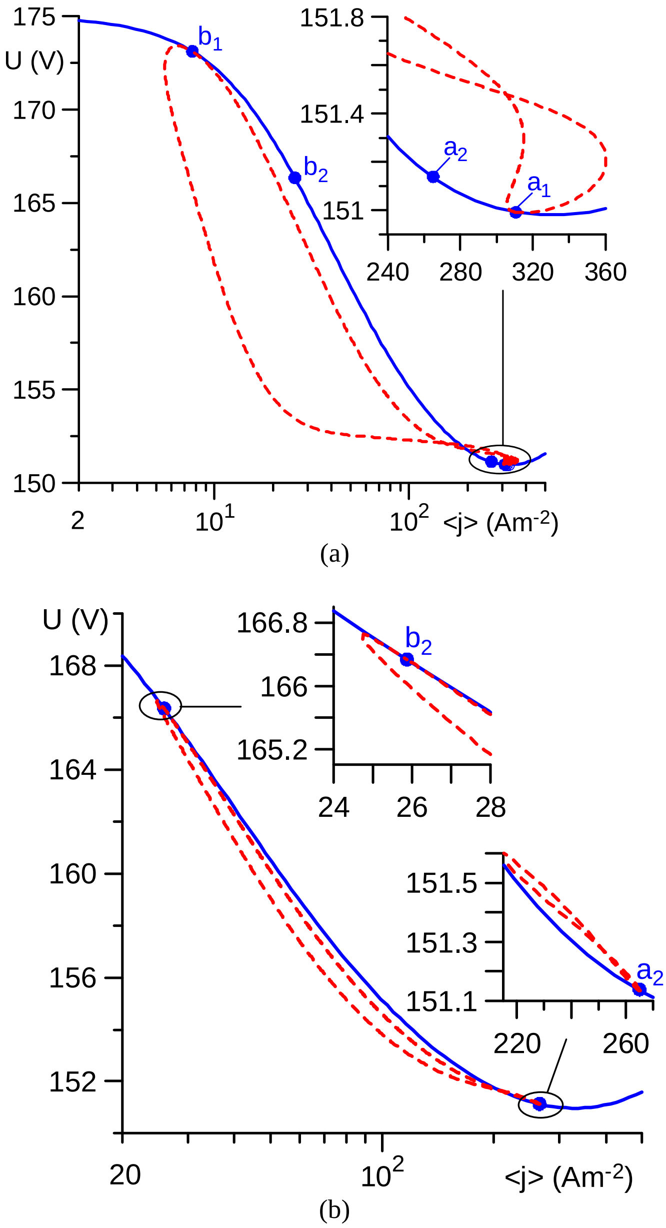

Figure 2: CVCs. R = 0.5 mm, diffusion losses neglected. (a) The first 2D mode and fundamental mode; (b) the second 2D mode and the fundamental mode.

The fundamental solution describes the abnormal discharge, the normal discharge, the subnormal discharge, and the Townsend discharge, and is similar to the solutions computed in, e.g. [6-11]. The first and second non-fundamental 2D modes are associated with patterns with an interior ring spot and, respectively, a spot at the centre and an interior ring spot. Only the fundamental mode was found at R = 0.5 mm.

Concluding remarks

Multiple steady-state solutions have been found in the framework of the simplest self-consistent

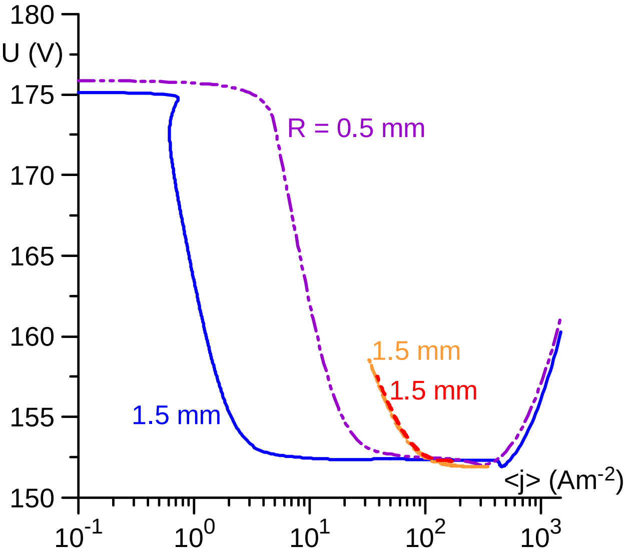

Figure 3: CVCs. Diffusion losses taken into account. Solid, two dot-dashed: fundamental mode. Dashed-dotted: the first non-fundamental 2D mode. Dashed: the second non-fundamental 2D mode.

model of DC glow discharge. When losses of the ions and the electrons due to diffusion to the wall are neglected, solutions describing nine different modes were detected in the case of a discharge tube of the radius R = 1.5 mm. One mode is 1D, is fundamental (i.e., exists at all values of the discharge current), and represents in essence the well-known solution of von Engel and Steenbeck. The other eight modes are 2D (axially symmetric) and exist in limited ranges of the discharge currents. Each 2D mode is constituted by two branches, one associated with a pattern comprising a spot at the centre of the cathode and the other with a pattern without a central spot. The branch with a spot at the centre of the cathode exhibits a well pronounced effect of normal current density. The number of existing 2D modes decreases with a decrease of the radius R of the discharge tube; there are only two modes at R = 0.5 mm.

When diffusion losses are taken into account, the number of multiple solutions is significantly reduced: only three 2D modes exist at R = 1.5 mm. One is the fundamental mode, comprising the Townsend, subnormal, normal and abnormal discharges. The non-fundamental modes describe a pattern with an interior ring spot and a pattern with a spot at the centre and an interior ring spot.

The patterns associated with multiple axially symmetric modes found in this work may be viewed as axially symmetric analogues of patterns observed in DC glow microdischarges, e.g. [1,2].

Future work should include finding multiple 3D solutions, an investigation of stability of different steady-state solutions, and eventually an introduction of more complex effects, such as the presence of multiple ion and/or neutral species, variations of the electron and heavy particle temperatures, nonlocality of the electron transport with the aim of explaining why patterns with multiple spots have been observed in xenon microdischarges and not in other discharges. Finding of multiple 3D solutions may be facilitated by bifurcation analysis in a similar way as it was done in this work while finding multiple 2D solutions.

Acknowledgments

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