Theory of Space-Charge Sheaths on Cathodes of Vacuum Arcs

M. S. Benilov and L. G. Benilova
Departamento de Física, Universidade da Madeira, 9000 Funchal, Portugal

***Abstract*- The model of a collisionless near-cathode space-charge sheath with ionization of atoms emitted by the cathode surface is considered. It was found that such sheath represents a double layer with a potential maximum, with the ions which are produced before the maximum returning to the cathode surface and those produced after the maximum escaping into the plasma. Distributions across the sheath have been calculated of ion, electron, and atomic densities, electrostatic potential and electric field. Also calculated have been integral parameters of the sheath. The results may be readily incorporated into models of near-cathode layers of discharges burning in cathode vapor. Besides, these results may be employed for qualitative analysis. In particular, the results indicate that the ion backflow coefficient in such discharges is at least 53%. As an example of application of the theory, calculation results on spots on copper cathodes of vacuum arcs are given and a favorable agreement with available experimental data is found.**

I. INTRODUCTION

It was realized long ago that distributions of potential in discharges burning in cathode vapor, such as vacuum arcs and low- to high-pressure arc discharges on cathodes made of volatile materials, may possess a maximum (potential hump). For the first time it was apparently hypothesized by Plyutto and co-workers [1] in order to explain the acceleration of ions towards the anode, observed in the experiment. Although by now most researchers seem to believe that the plasma acceleration in cathode jets is of a gas dynamic nature and associated with a plasma pressure gradient caused by very high pressures occurring in cathode spots (e.g., [2] and references therein), the question of potential hump in discharges burning in cathode vapor retains its significance.

A potential hump of a height approximately corresponding to the plasma temperature was revealed by modeling of the region of expansion of the cathode jet [3], [4, p. 255]. This hump was attributed to the fact that the local electron pressure gradient is quite high and must be partially compensated by a retarding electric field; otherwise the electron current would be too high. In essence, this is the same mechanism that causes negative anode voltage drop in arc discharges.

There are reasons to believe that another potential hump should exist in a close proximity of the cathode surface, in a region where (cold) atoms emitted by the surface are ionized. The ions in this region are still cold and can hardly move against electric field, in contrast to what happens in the hot plasma ball. Hence, the potential distribution in the region where ionization occurs should have a maximum, with the ions which are produced before the maximum returning to the cathode surface and those produced after the maximum escaping into the plasma.

It is of interest in this connection to try to develop a self-consistent model of near-cathode layer in discharges burning in cathode vapor which would describe this potential maximum. In was hypothesized in [5] that such maximum may appear in the model of a space-charge sheath if ionization of emitted atoms inside the sheath is taken into account.

This model is illustrated by Fig. 1. There must be an inflexion point in the potential distribution positioned between the maximum point and the plasma, which means that the sheath is actually a double layer.

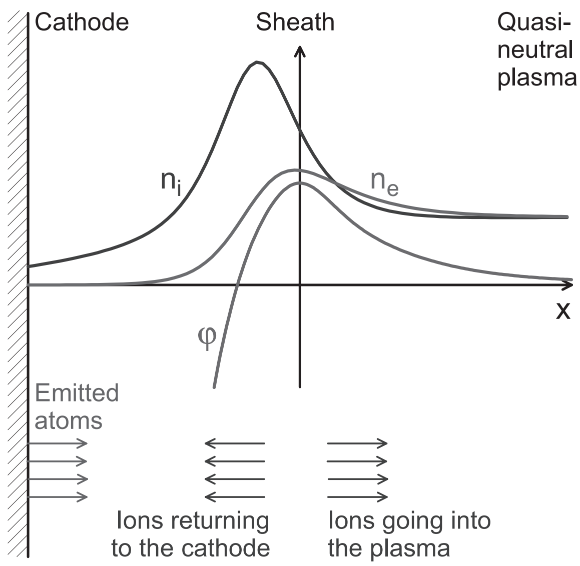

Fig. 1. Schematic of a sheath with ionization of emitted atoms

This model is treated in the present work. Since the motion of ions in the near-cathode space-charge sheaths in discharges burning in cathode vapor is rather collision-free than collision-dominated, the treatment is restricted to the case where the ions move without collisions in the sheath.

Bolotov and co-workers [6, 7] developed a quantitative model of potential hump in the cathode layer which has a number of similarities with the model

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of this work, and concluded that it offers explanation to a number of features exhibited by vacuum arcs. However, a self-consistent solution of the Poisson equation was not attempted and a linear distribution of electric field in the cathode layer was assumed instead.

The presence of a potential hump with ions generated at rest on both sides from the hump and moving away from it without collisions results in certain similarities between he present model and the model of Tonks and Langmuir of a collisionless positive column of a plane glow discharge enclosed by two parallel absorbing walls [8]. However, the potential hump in the Tonks and Langmuir model is of another nature (a consequence of symmetry) and its position is known (the axis of the discharge), in contrast to what happens in the present model.

II. THE MODEL AND NUMERICS

Let us introduce an axis x directed from the cathode surface into the plasma with the origin at point of the maximum of electrostatic potential as shown in Fig. 1. There is a flux of atoms emitted by the surface. As the atoms move into the plasma, they are gradually getting ionized and the variation of their number density n_a is governed by the equation $d(n_a v_a)dx = w$, where v_a is the average velocity of the atoms and w is the ionization rate.

The estimates [9] show that electrons emitted by the cathode transfer their energy to the plasma electrons, and those produce ionization. Then the ionization rate may be written as $w = k_i n_e n_a$, where k_i is the ionization rate coefficient (a known function of the plasma electron temperature) and n_e is the density of plasma electrons. The distribution of plasma electrons is Maxwellian with the electron temperature T_e being constant across the sheath and the electron density is related to the electrostatic potential through the Boltzmann distribution. The Poisson equation is written neglecting a small space charge contributed by the emitted electrons. The expression for the ion density is similar to the one in the Tonks-Langmuir model [8].

The above-described problem is transformed to dimensionless variables: normalized coordinate $\xi = x k_i n_e^{(0}/v_a$; normalized ion, electron, and atom densities, $N_i = n_i/n_e^{(0)}$, $N_e = n_e/n_e^{(0)}$, $N_a = n_a/n_a^{(0)}$; normalized electrostatic potential $\Phi = e(\varphi-\varphi^{(0)})/kT_e$; normalized electric field $E = -d\Phi/d\xi$. Here k is the Boltzmann constant, e is the electron charge, and the upper index *(0)* designates values of the corresponding parameters at the point of maximum of potential.

There are two dimensionless control parameters in the normalized problem. One is defined as $\alpha = v_a(m_i/kTe)^{1/2}\varepsilon_0 kTe/(n_a^{(0)}e^2)(k_i n_a^{(0}/v_a)^2$, where m_i is atomic mass and ε_0 is the dielectric constant. This parameter characterizes the squared ratio of the Debye length to the scale of variation of the atomic density, evaluated in terms of quantities at the point of maximum of potential. The other dimensionless control parameter is the sheath voltage normalized by kT_e/e.

While solving the above-described problem numerically, it is convenient to treat the atomic density at the point of maximum, $n_a^{(0)}$, as a given parameter, and the atomic density at the cathode surface, n_{aw}, as a calculation result. Then the solution may be found first in the region beyond the potential maximum, $\xi \geq 0$, and then in the region between the maximum point and the cathode, $\xi \leq 0$. The problem in the region $\xi \geq 0$ represents a two-point boundary-value problem for two equations, one of these equations being (ordinary) differential and the other integrodifferential. This problem is not quite trivial from the mathematical point of view and a special care was employed in order to show that it is solvable and its solution is unique. The problem in the region $\xi \leq 0$ is essentially an initial-value problem and poses no difficulties.

III. RESULTS AND DISCUSSION

Examples of calculated distributions of normalized parameters in the sheath are shown in Fig. 2. The solid and dashed lines in Fig. 2 represent results obtained on numerical grids with different steps, $h = 10^{-2}$ and $h = 10^{-3}$, respectively. There is a difference of up to 10% between the two sets of results, which gives an idea of the overall accuracy of the numerical date reported in this paper.

One can see that the numerical results confirm the physical picture hypothesized in the Introduction. There is a maximum in the distribution of potential and, consequently, of the electron density. There is a maximum also in the distribution of the electric field, an indication of a double layer. The ion density is non-monotonic as well, with a maximum positioned at a negative ξ.

The density (and flux) of atoms emitted by the cathode surface remain unaltered in the vicinity of the cathode, where the plasma is strongly negative and from where the electrons are repelled by the sheath electric field, so no ionization occurs. Further away from the cathode, the electron density becomes appreciable and atoms start getting ionized: N_a starts decreasing. As ξ increases, the atomic density decreases rather fast and soon becomes negligible, i.e., the plasma becomes fully ionized.

At $\alpha = 0.1$, the ion density in the region of the potential hump is relatively close to the electron density, so the plasma is not very far from quasi-neutrality here. With increasing α, the ion density in the region of the potential hump increases and at large α considerably exceeds the electron density.

In addition to distributions of parameters across the sheath, also integral parameters have been calculated which are essential for understanding and modeling of plasma-cathode interaction in discharges burning in cathode vapor, in particular, $\tau = v_a(m_i/kTe)^{1/2} n_a^{(0)}/n_e^{(0)}$ the ratio of characteristic fluxes of the atoms and the ions; Φ_∞ the dimensionless height of the potential hump; N_{aw} the ratio of the atomic density at the cathode surface to the atomic density at the potential maximum; Ψ_{iw}, $\Psi_{i\infty}$ the dimensionless average potential energies with which ions are produced before and after potential maximum, respectively. τ, Φ_∞, and $\Psi_{i\infty}$ represent functions of a single control parameter α.

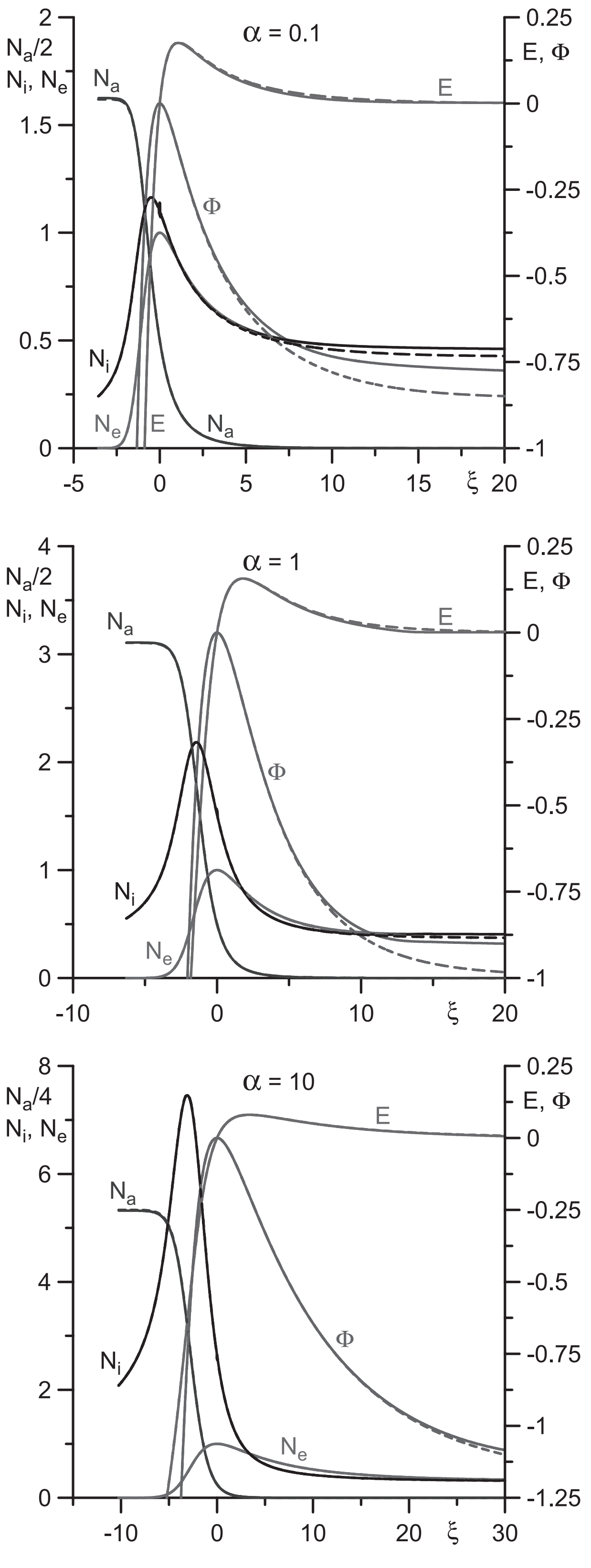

Fig. 2. Distribution of parameters across the sheath.

N_{aw} and Ψ_{iw} depend also on the dimensionless sheath voltage, however, this dependence is weak under conditions of practical interest and one can consider N_{aw} and Ψ_{iw} as functions of the single parameter α as well.

The range of variation of parameter τ is rather narrow: as α increases from very low to very high values, τ decreases from 0.487 to 0.451, i.e., by about 8%. Φ_∞ decreases more significantly, although not dramatically: from -0.85 to -1.26. Ψ_{iw} also decreases significantly but not dramatically, from -0.56 for $\alpha = 0.1$ to -0.89 for $\alpha = 10$. $\Psi_{i\infty}$ increases from -0.14 for small α to very small negative values for large α. N_{aw} varies quite strongly, from 2.11 for very small α to 21.3 for $\alpha = 10$. With increase of α, the maximum of the electric field in normalized variables is shifted further away from the potential hump into the plasma and the height of the maximum decreases.

In order to make the obtained results practicable, one needs to relate α to a parameter evaluated in terms of quantities at the cathode surface. An appropriate parameter is $\alpha_w = v_a(m_i/kTe)^{1/2}\varepsilon_0 kTe/(n_{aw}e^2)(k_i n_{aw}/v_a)^2$. The relation between α_w and α reads $\alpha_w(\alpha) = \alpha N_{aw}(\alpha)$ and may be readily evaluated with the use of the numerical data obtained.

The above-described calculation data on parameters τ, Φ_∞, N_{aw}, Ψ_{iw}, and $\Psi_{i\infty}$ can be used for evaluation of a number of quantities which are essential for understanding and modeling of plasma-cathode interaction in vacuum arcs: the height of the potential hump, the average kinetic energy of ions bombarding the cathode surface, the average kinetic energy of ions leaving the sheath, the electric field at the cathode surface. In particular, it follows that the height of the potential hump is within the range $(0.85\ldots1.26)kT_e/e$. This value is insufficient to explain the observed velocities of ions in cathode jets of vacuum arcs, in agreement with the belief of many researchers that the main contribution to acceleration of ions is given by the plasma pressure gradient. The ion backflow coefficient cannot be below 53%.

These results may be incorporated into models of near-cathode layers and cathode spots in discharges burning in cathode vapor. As an example, parameters of spots on copper cathodes of vacuum arcs, calculated by means of a spot model with represents a development of the model [10] and makes use of the above results, are shown in Table 1. Here I is the current per spot, U is the sheath voltage, g is the erosion rate, T_* and T_{max} are local temperatures of the cathode surface at the spot edge and the spot center, and U_h is the so-called heating voltage. The average temperature of the cathode surface outside spots in these calculations was set equal to 1000K. It should be stressed that the model contains no empirical/tunable parameters.

For the current per spot in the range between 40 and 200A, the model predicts the near-cathode voltage between 15 and 20V, in agreement with the usually cited experimental values.

TABLE 1. PARAMETERS OF SPOTS ON COPPER CATHODE

I(A)	U(V)	g(μg/C)	T_*(K)	T_{max}(K)	T_{e*}(K)	U_h(V)
193	15	56	4075	4225	21955	2.6
78	17	36	4020	4205	28685	3.1
36	20	20	3980	4190	39323	3.6

The erosion rate g estimated in the present work refers to the so-called ion erosion, which is erosion due to flux of ions leaving the near-cathode layer for the bulk plasma; the erosion due to formation of macroparticles is not considered in this work. Experimental values of ion erosion rate for copper are in the range 33-39μg/C [11]. Values of the erosion rate predicted by the model are between 56 and 20μg/C, in a reasonable agreement with the experimental values. This agreement supports the model of a space-charge sheath on evaporating cathodes, described in this work.

The predicted spot temperature is somewhat higher than 4000, the electron temperature in the near-cathode layer is between 2 and 4eV. These values also seem to be reasonable enough.

IV. CONCLUSIONS

Numerical results confirm the concept of a near-cathode space-charge sheath with ionization of atoms emitted by the cathode surface being a double layer with a potential hump: the mathematical problem is solvable and its solution is unique.

The results may be readily incorporated into theoretical and numerical models of near-cathode layers of discharges burning in cathode vapor. Besides, the results may be employed for qualitative analysis.

In particular, the results indicate that the ion backflow coefficient in such discharges is at least 53%. The height of the potential hump is within the range $(0.85\ldots1.26)kT_e/e$. This value is insufficient to explain the observed velocities of ions in cathode jets of vacuum arcs, in agreement with the belief of many researchers that the main contribution to acceleration of ions is given by the plasma pressure gradient.

As an example of application of the theory, calculation results on spots on copper cathodes of vacuum arcs are given and a favorable agreement with available experimental data is found.

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E-mail of authors: benilov@uma.p