Summary of the Theory of Interaction of High-Pressure Arc Plasmas with Thermionic Cathodes

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Abstract

This is a slightly modified version of Appendix to the paper [1]. Here, a summary is given of equations of the theory of interaction of thermionic cathodes with high-pressure plasmas composed of atoms of a single species, ions of a single species, and electrons. Detailed presentations of different aspects of the theory can be found in the original works cited below.

1 Model of nonlinear surface heating

Let us consider a thermionic cathode of a high-pressure arc discharge. Joule heat generation inside the cathode body is assumed to be negligible, the thermal conductivity κ of the cathode material is assumed to be a known function of its temperature: $\kappa = \kappa(T)$. The base of the cathode is maintained at a fixed temperature T_c by external cooling and the rest of the cathode surface is in contact with the plasma or the cold gas and is heated or, respectively, cooled.

A steady-state temperature distribution in the cathode body is governed by the equation of thermal conduction

$$\nabla \cdot (\kappa \nabla T) = 0. \tag{1}$$

The boundary condition at the base of the cathode reads

$$T = T_c. \tag{2}$$

The density q of the net energy flux from the plasma or the cold gas to the cathode surface is evaluated as difference between the density of the energy flux from the plasma to the cathode surface, q_p, and the density of radiation losses of energy by the cathode surface: $q = q_p - \varepsilon\sigma T_w^4$, where ε is the hemispheric total emissivity of the cathode material (a known function of the surface temperature T_w) and σ is the Stefan-Boltzmann constant.

It is assumed that q_p and the density j of electric current from the plasma to the cathode surface are functions of the local temperature T_w of the cathode surface and of the near-cathode voltage drop U: $q_p = q_p(T_w, U)$, $j = j(T_w, U)$. (One of conditions of validity of this assumption is that energy flux coming from the arc plasma to the current-collecting part of the cathode surface be generated in a thin near-cathode plasma layer which is independent of the bulk plasma.) The near-cathode voltage drop U is assumed to be a given parameter which is the same at all points of the current-collecting part of the surface.

Under the above assumptions, the boundary condition at the part of the cathode surface that is in contact with the arc plasma and with the cold gas reads

$$\kappa \frac{\partial T}{\partial n} = q\left(T_w, U\right), \tag{3}$$

where n is a direction locally orthogonal to the cathode surface and directed outside the cathode.

In the framework of the above-described approach, a description of the arc-cathode interaction may be constructed in two steps. At the first step, the (one-dimensional) problem describing the current transfer across the near-cathode plasma layer is solved and all parameters of the layer are determined as functions of T_w and U. In particular, densities of the energy flux and of the electric current from the plasma to the current-collecting part of the cathode surface, $q_p = q_p\left(T_w, U\right)$ and $j = j\left(T_w, U\right)$, are determined. At the second step, the nonlinear thermal-conduction problem (1)-(3) is solved.

A detailed presentation of the model of nonlinear surface heating can be found, e.g., in [2]. Here, we emphasize only that what is specified in the framework of this approach is not a distribution of the energy flux from the plasma over the cathode surface but rather a dependence of the energy flux density on the local surface temperature, this temperature being unknown *apriori*. On having solved the thermal-conduction problem (1)-(3), one will have complete information on a temperature distribution in the cathode and also on a distribution of the energy flux and electric current over the cathode surface. Integrating the latter, one will find the arc current I corresponding to a value of U being considered.

Nowadays, the second step of the above-described procedure in most cases does not pose major difficulties, at least as far as solutions are concerned describing the diffuse mode of current transfer. The first step is described in the following section.

2 Near-cathode plasma layer

In this section, a summary is given of equations describing the near-cathode layer of plasmas composed of atoms of a single species, ions of a single species, and electrons. A generalization of the model for the case of multiple plasma-producing species can be found in [1].

The near-cathode plasma layer comprises a number of sub-layers, of which the most important are a space-charge sheath, which is adjacent to the cathode surface, and an ionization layer, which is adjacent to the sheath. The ion flux to the cathode is generated in the ionization layer. In the sheath, the ions moving to the cathode and electrons emitted from the cathode are accelerated.

The space-charge sheath is considered as collisionless for ions. The number density of flux of ions to the cathode surface, being equal to the density of flux of ions from the ionization layer to the sheath edge, is evaluated as

$$J_i = n_{is} v_s, \qquad v_s = \sqrt{\frac{k\left(T_h + T_e\right)}{m_i}}, \tag{4}$$

where n_{is} is the ion (or electron) density at the sheath edge, v_s is the Bohm velocity, T_e is the temperature of electrons which is assumed to be constant across the ionization layer and the sheath, T_h is the temperature of heavy particles (ions and neutral atoms) which is assumed to be constant across the ionization layer and the sheath and equal to the temperature T_w of the cathode surface, m_i, m_a and m_e here and further are masses of the ion, the atom and the electron.

The number density of flux of plasma electrons which come to the cathode surface from the ionization layer after having overcome the retarding electric field in the space-charge sheath is

$$J_e = \frac{1}{4} n_{is} \sqrt{\frac{8kT_e}{\pi m_e}} \exp\left(-\frac{eU_D}{kT_e}\right), \tag{5}$$

where U_D is the voltage drop in the sheath.

J_{em} the electron emission flux from the cathode surface is evaluated by means of the Richardson-Schottky formula. The electric field at the cathode surface, involved in this formula, is obtained by solving the Poisson equation in the sheath jointly with a kinetic equation describing the motion of ions and the Boltzmann distribution for plasma electrons (the space charge of emitted electrons is neglected) and reads

$$E_w = \sqrt{\frac{2n_{is}kT_h}{\epsilon_0}} \left[\frac{v_+^3 - v_-^3}{6u_i^3} - \frac{4}{3} - 2\beta + \beta \exp\left(-\frac{eU_D}{kT_e}\right)\right]^{1/2}, \tag{6}$$

where

$$u_i = \sqrt{\frac{kT_h}{m_i}}, \quad \beta = \frac{T_e}{T_h}, \quad v_\pm = \left[\left(v_s \pm u_i\right)^2 + \frac{2eU_D}{m_i}\right]^{1/2}. \tag{7}$$

The densities of net electric current and of plasma-related net energy flux to the cathode surface are

$$j = e\left(J_i + J_{em} - J_e\right), \quad q_p = jU - \frac{j}{e}\left(A + 3.2kT_e\right). \tag{8}$$

On the edge (plasma side) of the ionization layer, the ionization equilibrium is assumed. The local ion (or electron) density $n_{i\infty}$ and atomic density $n_{a\infty}$ are evaluated, for given temperatures T_h and T_e and plasma pressure p, with the use of the Saha equation. The variation of the charged particle density across the ionization layer is given by the formula

$$\frac{n_{is}}{n_{i\infty}} = \frac{\alpha C_2 \sqrt{1+\beta}}{C_2 + 2\alpha C_2 \sqrt{1+\beta} + \alpha^2 \sqrt{1+\beta}}. \tag{9}$$

Here C_2 is a dimensionless coefficient defined by Eq. (37) of Ref. [3], which depends on β and $\gamma = n_{i\infty}/n_{a\infty}$ and varies for $\beta \geq 1$ between approximately 0.67 and 1 (see Fig. 7 of [4]). α is the ratio of the ionization length to the mean free path for ion-atom collisions defined by the formula

$$\alpha = \sqrt{\frac{2}{3} \frac{C_{ia} Q_{ia}}{k_i}}, \quad C_{ia} = \sqrt{\frac{8kT_h}{\pi} \left(\frac{1}{m_i} + \frac{1}{m_a}\right)}, \tag{10}$$

where Q_{ia} and k_i are the average cross section for momentum transfer in elastic collisions ion-atom and the rate constant of ionization of atoms for the gas being considered. Note that C_{ia} has the meaning of average relative speed of ions and atoms; in the case of a plasma produced in a pure monoatomic gas being under consideration in this section, $m_i \approx m_a$ and the second equation in Eq. (10) coincides with the corresponding expression in [4].

The voltage drop in the ionization layer and the total voltage drop in the near-cathode layer are evaluated as

$$U_i = \frac{kT_e}{e} \ln \frac{n_{i\infty}}{n_{is}}, \quad U = U_D + U_i. \tag{11}$$

The equation of balance of the electron energy in the ionization layer reads

$$J_{em} \left(2kT_w + eU_D - \Delta A \right) + \frac{e \left(J_{em} - J_e \right) + j}{2} U_i$$
$$= J_e \left(2kT_e + eU_D - \Delta A \right) + 3.2 \frac{j}{e} kT_e + J_i E, \tag{12}$$

where ΔA is the Schottky correction to the work function and E is the ionization energy.

The above-described relationships represent a complete set of equations which allows one to determine all parameters of the near-cathode plasma layer for a given plasma-producing gas, plasma pressure, and work function of the cathode material as functions of T_w and U. In particular, one can determine functions $q_p \left(T_w, U \right)$ and $j \left(T_w, U \right)$. A detailed presentation of the model can be found in [2, 5, 6].

Note that difficulties arise in solving the above-described equations under conditions at which there is practically no plasma adjacent to the cathode. Therefore, these equations are solved at T_w exceeding certain temperature value which is typically 500 K; q_p and j are set equal to 0 at lower T_w. (It should be emphasized that this comment has purely a technical sense and the value $T_w = 500$ K should not be identified with the temperature separating the cathode surface in contact with the plasma from that in contact with the cold gas.)

All the papers cited below are available at
http://fisica.uma.pt/ingles/pessoal/Mikhail_Benilov/lista.html

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