UNIVERSIDADE da MADEIRA

Electromagnetism

Series of problems 4

1. A typical laboratory experiment wire is made of copper and has a radius of 0.815 mm. Calculate the drift velocity (or average velocity) of the electrons in that wire. If it carries a current of 1 A, assume one free electron per atom. Note: N_a = Avogadro's number $= 6,02 \times 10^{23}$ atoms / mol; $\rho = 8,93\,\mathrm{g/cm^3}$; M = copper molar mass $= 63,5\,\mathrm{g/mol}$.

2. In a given particle accelerator, a current of 0.5 mA is carried by a 5 MeV proton beam having a radius of 1.5 mm. (Note: mass of a proton is 1.67×10^{-27} kg.)
   (a) Find the numerical density of protons in the beam.
   (b) If the beam hits a target, how many protons hit the target in 1 s?

3. The current density in a cylindrical wire of radius $R = 2.0$ mm is uniform across the wire section and is $j = 2.0 \times 10^5\,\mathrm{A/m^2}$.
   (a) What is the current through the outer portion of the wire between the radial distances $R/2$ e R?
   (b) Assume that the current density across the section varies with the radial distance r as $j = ar^2$, where $a = 3,0 \times 10^{11}\,\mathrm{A/m^4}$ and r is in meters. What is now the current through the same outer portion of the wire?

4. Consider current density
$$\mathbf{j} = -10^4 \begin{bmatrix} \sin(2x)\exp(-2y)\,\mathbf{a}_x + \\ \cos(2x)\exp(-2y)\,\mathbf{a}_y \end{bmatrix} \mathrm{kA\,m^{-2}}$$
   (a) Find the current running through the plane $y = 1$ in the direction $\mathbf{a}_y$ in the region $0 < x < 1$, $0 < z < 2$.
   (b) Find the current that exits the region $0 < x < 1$, $2 < z < 3$, $0 < y < 1$ by integrating $\mathbf{j} \cdot d\mathbf{S}$ on the surface of the cube.

5. Find the current running through the portion of the plane $y = 0$ defined by $-0,1 \leq x \leq 0,1$ m and $-0,002 \leq z \leq 0,002$ m if $\mathbf{j} = 10^2\,|x|\,\mathbf{a}_y \quad (\mathrm{A\,m^{-2}})$.

6. Find the current running through the portion of the plane $x = 0$ defined by $-\frac{\pi}{4} \leq y \leq \frac{\pi}{4}$ m and $-0,01 \leq z \leq 0,01$ m if $\mathbf{j} = 100\cos(2y)\,\mathbf{a}_x \quad (\mathrm{A\,m^{-2}})$.

7. Let $j = 10^3 \sin(\theta)\, a_r \quad (\mathrm{A\,m^{-2}})$ (in spherical coordinates). Find the current running through the spherical surface $r = 0.02$ m.

8. Calculate the resistance of an aluminum cylinder that is 10.0 cm long and $2.00 \times 10^{-4}\,\mathrm{m^2}$ in cross-sectional area. (The resistivity of aluminum is $2,82 \times 10^{-8}\,\Omega\,\mathrm{m}$). Repeat the calculation for a cylinder of the same size and made of glass with a resistivity $3,0 \times 10^{10}\,\Omega\,\mathrm{m}$.

9. The resistivity of a nickel-chromium wire is $1.5 \times 10^{-6}\,\Omega\,\mathrm{m}$.
   (a) Calculate the resistance per unit length of wire that has a radius of 0.321 mm.
   (b) If we apply a potential difference of 10 V to a 1.0 m long nickel-chromium wire, what is the current in the wire?

10. Coaxial cables are used extensively in cable television and other electronic applications. A coaxial cable consists of two concentric cylindrical conductors. The region between the conductors is completely filled with silicone, and the current lost through the silicone in the radial direction is undesirable (the cable is designed to conduct current along its length only). The radius of the inner conductor is $a = 0.500$ cm, the radius of the outer conductor is $b = 1.75$ cm, and the length $L = 15.0$ cm. Calculate the resistance of silicone between the two conductors, knowing that the resistivity of silicone is $640\,\Omega\,\mathrm{m}$.

Compare the resistance with that of the inner conductor (assume it is made of copper whose resistivity is 1.7 $\times 10^{-8}\,\Omega\,\mathrm{m}$).

11. An electric heater is constructed by applying a potential difference of 120 V to a nickel-chromium wire that has a total resistance of 8.00 Ω. Find the current carried by the wire and the power of the heater.

12. A nickel chromium wire is commonly used as the heating element in electrical equipment. One of these 1.0 m long wires is used at the bottom of an oven and can withstand a maximum current of 16 A when a potential difference of 120 V is applied to the ends of the wire. If the resistivity of the wire is $1.0 \times 10^{-6}\,\Omega\,\mathrm{m}$:

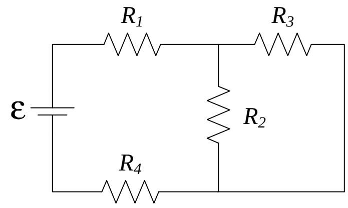
    (a) What is the radius of the wire?

    (b) What is the power used by the oven?

13. A 100 W lamp is left lit in an outdoor pantry to prevent ink from freezing. The 100 W correspond to the dissipated power in the lamp filament, which is a simple resistance. If electricity costs $8 cents/\,\mathrm{kW\,h}$, how much does it cost to keep the lamp on for 3 months during the winter?

14. The heating element in the rear window defroster of a Mazda RX-7 has a resistance of 3.00 Ω. The element plugs directly into the car's 12.0 V battery. How much heat is produced in the element in 10.0 min and how much ice can it melt? (The energy required to melt 1 g of ice is 335 J).

15. Consider the schematic circuit that contains an ideal battery and four resistors:

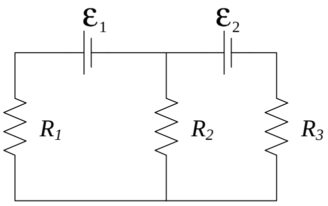


Determine all currents in the circuit knowing that:

$$\begin{aligned} \varepsilon &= 12\,\mathrm{V}; R_1 = 20\,\Omega; R_2 = 20\,\Omega; \\ R_3 &= 30\,\Omega; R_4 = 8,0\,\Omega \end{aligned}$$

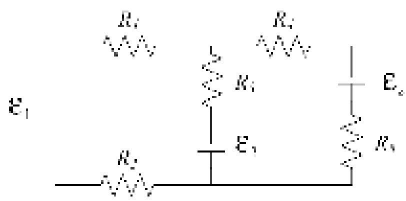
16. Find the currents in the following circuit, knowing that:

$$\begin{aligned} \varepsilon_1 &= 6,00\,\mathrm{V}; \varepsilon_2 = 12,0\,\mathrm{V}; \\ R_1 &= 100,0\,\Omega; R_2 = 10,0\,\Omega; R_3 = 80,0\,\Omega \end{aligned}$$



17. Find the currents in the following circuit, knowing that:

$$\begin{aligned} \varepsilon_1 &= 3,0\,\mathrm{V}; \varepsilon_2 = 6,0\,\mathrm{V}; R_3 = 4,0\,\Omega; \\ R_1 &= R_2 = R_4 = R_5 = 2,0\,\Omega \end{aligned}$$



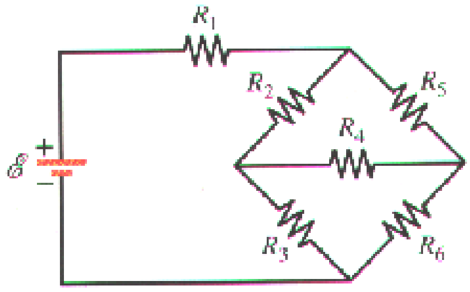
Solutions:

1) $v_d = 3,54 \times 10^{-5}\,\mathrm{m\,s^{-1}}$; 2a) $n = 1,43 \times 10^{13}$ protons/ m^3; 2b) $N = 3,13 \times 10^{15}$ protons; 3a) $I = 1,9\,\mathrm{A}$; 3b) $I = 7,1\,\mathrm{A}$; 4a) $I = -1231\,\mathrm{kA}$; 4b) $I = 0\,\mathrm{A}$; 5) $I = 4\,\mathrm{mA}$; 6) $I = 2\,\mathrm{A}$; 7) $I = 3,95\,\mathrm{A}$; 8) $R_{al} = 1,41 \times 10^{-5}\,\Omega$; $R_{vi} = 1,5 \times 10^{13}\,\Omega$; 9a) $R = 4,6\,\Omega\,\mathrm{m}^{-1}$; 9b) $I = 2,2\,\mathrm{A}$; 10) $R = 851\,\Omega$; $R_{Cu} = 3,2 \times 10^{-5}\,\Omega$; 11) $I = 15,0\,\mathrm{A}$; $P = 1,80 \times 10^3\,\mathrm{W}$; 12a) $r = 0,20\,\mathrm{mm}$; 12b) $P = 1920\,\mathrm{W}$; 13) Cost: 1728 cents; 14) Heat=energy: $2,88 \times 10^4\,\mathrm{J}$; $m_{gelo} = 86,0\,\mathrm{g}$; 15) $i_1 = 0,3\,\mathrm{A}; i_2 = 0,18\,\mathrm{A}; i_3 = 0,12\,\mathrm{A}$; 16) $i_2 \approx -74\,\mathrm{mA}; i_3 \approx 141\,\mathrm{mA}; i_1 \approx 67\,\mathrm{mA}$; 17) $i_1 = -0,5\,\mathrm{A}; i_3 = -0,25\,\mathrm{A}; i_4 = -0,25\,\mathrm{A}$.

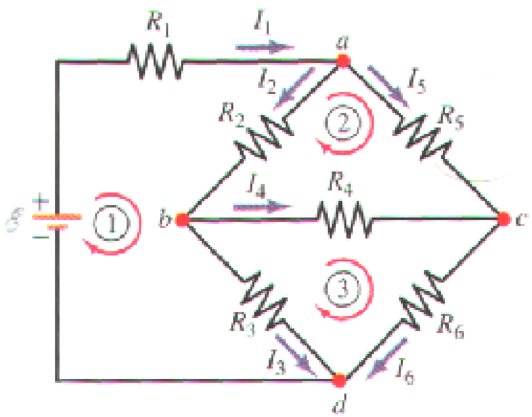
Additional solved problem:

- Assume that the electromotive forces and resistances of the schematic circuit are known. Express

the linear equations that can be solved to find the values of all currents.



Solution:
Let's consider the currents represented as follows:



In this case our equations are written:

mesh 1 : $\varepsilon = R_1 i_1 + R_2 i_2 + R_3 i_3$

mesh 2 : $0 = R_5 i_5 + R_4 (-i_4) + R_2 (-i_2)$

mesh 3 : $0 = R_4 i_4 + R_6 i_6 + R_3 (-i_3)$

node a : $i_1 = i_2 + i_5$

node b : $i_2 = i_3 + i_4$

node c : $i_4 + i_5 = i_6$

node d : $i_3 + i_6 = i_1$

We have a system of 6 equations with 6 unknowns. This system can therefore be solved.