Cadeira: **Estudo do Meio Físico-Natural I**		**Época:** **Normal**
Ano lectivo: 2013/2014 (1º Semestre)	**TESTE 1** (2013/11/13)	**Duração:** 2 horas
Nome: ______________________________	Número: __________	Curso: **Educação Básica**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	T

Some formulas:

Relative variation (in percentage): $\frac{x_f - x_i}{x_i} \times 100\%$

Norm of a vector: $A = \sqrt{A_x^2 + A_y^2}$

Uniform motion: $v = v_0$; $x = x_0 + v_0 t$

Uniformly varied motion: $v = v_0 + at$; $x = x_0 + v_0 t + \frac{1}{2}at^2$

$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$; $\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$; $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\text{opposite side}}{\text{adjacent side}}$

Gravitic force: $F_g = G\frac{m_1 m_2}{r^2}$

Friction force: $F_{ae\,\max} = \mu_e mg$; $F_{ac} = \mu_c mg$

Center of mass: $\vec{r}_{\text{CM}} = \sum_{i=1}^{N} \frac{m_i \vec{r}_i}{m_i} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots}{m_1 + m_2 + \cdots}$

Volumetric mass density: $\rho = \frac{m}{V}$

Pressure: $p = \frac{F}{A}$ Pressure at depth h: $p = p_0 + \rho g h$

Continuity equation: $A_1 v_1 = A_2 v_2$ Bernoulli equation: $p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

Some constants and conversion factors: $G = 6,67 \times 10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}$; $g = 9,8\,\text{m}\,\text{s}^{-2}$; $\rho_{H_2O} = 1\,\text{g}\,\text{cm}^{-3} = 10^3\,\text{kg}\,\text{m}^{-3}$; atmospheric pressure $= 1.013 \times 10^5\,\text{Pa}$. $1\,\text{l} = 1\,\text{dm}^3$.

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1. [0.5] Convert 13 inches (13 in) in centimeters, knowing that $1\,\text{in} = 2,54\,\text{cm}$.

   Solution: $13\,\text{in} \times \frac{2.54\,\text{cm}}{1\,\text{in}} = 0.3302\,\text{m} \approx 33\,\text{cm}$.

2. [0.75] Consider the following expression, where F is a force, m a mass, v a velocity and r a radius.

   $$F = m\frac{v^2}{r}$$

   Check if the expression is dimensionally correct.

   Solution:

   $$\text{LMT}^{-2} = \text{M}\frac{\left(\text{LT}^{-1}\right)^2}{\text{L}} = \text{LMT}^{-2}.$$

   So the expression is dimensionally correct.

3. [1] Write the following numbers in scientific notation:

   $46380 =$

   $0,002133 =$

   $9934 \times 10^{-2} =$

   $0,013 \times 10^{-5} =$

   Solution: $4,6380 \times 10^4$; $2,133 \times 10^{-3}$; $9,934 \times 10^1$; $1,3 \times 10^{-7}$.

4. [0.5] Determine the order of magnitude of $0,327$.

Solution:

$$0,327 = \underbrace{3,27}_{>\sqrt{10}} \times 10^{-1} \sim 10^{1} \times 10^{-1} = 1$$

5. [1] How many orders of magnitude the number $0,014$ is less than 0.052?

Solution:

$$\begin{aligned} 0,014 &= \underbrace{1,4}_{<\sqrt{10}} \times 10^{-2} \sim 10^{-2} \\ 0,052 &= \underbrace{5,2}_{>\sqrt{10}} \times 10^{-2} \sim 10^{1} \times 10^{-2} = 10^{-1} \end{aligned}$$

To compare the orders of magnitude we wil calculate the ratio between the two

$$\frac{10^{-2}}{10^{-1}} = 10^{-1}$$

The number $0,014$ has one order of magnitude less than the number $0,052$.

6. [0,5] A skateboard saw its price reduced by 35%. The old price of the skateboard was 110 €. Determine the new price of the skateboard.

Solution: The reduction in price can be obtained by: $\frac{35}{100} \times 110$ € $= 38.5$ €.

So, the new price is: 110 € -38.5 € $= 71.5$ €.

7. [0,5] A quantity increased to 6/5 of its initial value. How much was the relative variation (in percentage)?

Solution: $x_f = \frac{6}{5}x_i \longrightarrow \frac{x_f - x_i}{x_i} \times 100\% = \frac{\frac{6}{5}x_i - x_i}{x_i} \times 100\% = \frac{\frac{6}{5} - 1}{1} \times 100\% = \frac{\frac{6}{5} - \frac{5}{5}}{1} \times 100\% = \frac{1}{5} \times 100\% = 20\%$

8. [1] An average person sneezes 3 times per day. Estimate the order of magnitude of the total number of times that the person has sneezed by the age of 60.

Solution:

$$\frac{3 \text{ times}}{1 \text{ day}} \times \frac{365 \text{ day}}{1 \text{ year}} \times 60 \text{ year} = 3 \times 365 \times 60 \text{ times}$$

Since we are interested in orders of magnitude we estimate the order of magnitude of each factor: $3 \sim 1$; $365 \sim 10^3$; $60 \sim 10^2$. Hence, the person has sneezed approximately

$$1 \times 10^3 \times 10^2 \text{ times} = 10^5 \text{ times}$$

Out of curiosity, without rounding the value would be:

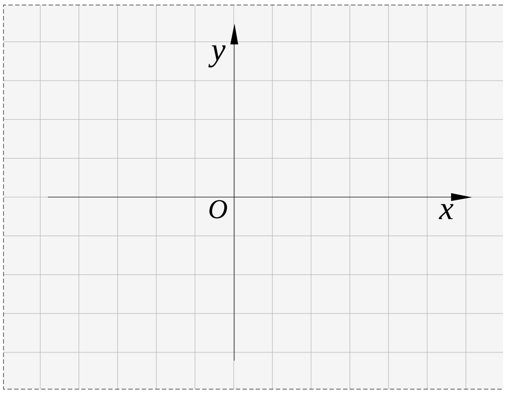
$$\frac{3 \text{ times}}{1 \text{ day}} \times \frac{365 \text{ day}}{1 \text{ year}} \times 60 \text{ year} = 3 \times 365 \times 60 \text{ times}$$
$$65700.0 \text{ times} \approx 6,57 \times 10^4 \text{ times}$$

That is, our estimate showed a deviation of about $\left|\frac{6{,}57 \times 10^4 - 10^5}{6{,}57 \times 10^4}\right| \times 100\% = 52,207\% \approx 52\%$.

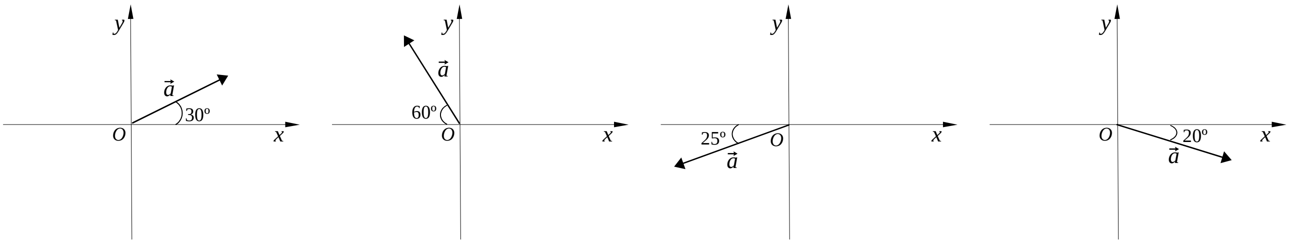
9. [0,5] Consider the vectors $\mathbf{A} = (2;4)$ and $\mathbf{B} = (5;7)$. Determine the components of the vectors $\mathbf{S} = \mathbf{A} + \mathbf{B}$ and $\mathbf{D} = \mathbf{A} - \mathbf{B}$.

Solution: $\mathbf{S} = \mathbf{A} + \mathbf{B} = (2;4) + (5;7) = (2+5;4+7) = (7;11)$. $\mathbf{D} = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) = (2;4) + (-5;-7) = (-3;-3)$.

10. [0, 75] Draw the vectors: $\mathbf{L} = (5, 3)$, $\mathbf{K} = (-3, -2)$ and $\mathbf{M} = (2, -1)$ in the next figure



11. [2] Knowing that the magnitude of the vector $\vec{a}$ represented in the figure is equal to 10.00, write the components a_x and a_y for each case.



Solution:

$$\text{(a)}: \begin{cases} a_x = 10.00 \cos 30\,° = 8.66 \\ a_y = 10.00 \sin 30\,° = 5.00 \end{cases}$$

$$\text{(b)}: \begin{cases} a_x = 10.00 \cos 120\,° = 10.00 \cos(-240\,°) = -5.00 \\ a_y = 10.00 \sin 120\,° = 10.00 \sin(-240\,°) = 8.66 \end{cases}$$

$$\text{(c)}: \begin{cases} a_x = 10.00 \cos 205\,° = 10.00 \cos(-155\,°) = -9.06 \\ a_y = 10.00 \sin 205\,° = 10.00 \sin(-155\,°) = -4.23 \end{cases}$$

$$\text{(d)}: \begin{cases} a_x = 10.00 \cos(-20\,°) = 9.40 \\ a_y = 10.00 \sin(-20\,°) = -3.42 \end{cases}$$

12. [0, 5] The position of a particle relative to time is given by $x = 0.3t^3 + 0.4t^2 + 0.5$ (SI). Determine the position for $t = 2\,\text{s}$.

Solution:

$$x(2\,\text{s}) = 0.3\,(2)^3 + 0.4\,(2)^2 + 0.5 = 4.5\,\text{m}$$

13. [1] A car is traveling at a constant speed of $120\,\text{km}\,\text{h}^{-1}$ in rectilinear motion. Suddenly the driver sees an obstacle in the road and to avoid a collision he presses the brakes, this makes the car start to reduce its speed at a constant rate of $-3\,\text{m}\,\text{s}^{-2}$.

(a) How much time does the car need to stop?

(b) What distance the car travels until it stops after starting to brake?

Solution:

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \rightarrow \Delta x = x - x_0 = v_0 t + \frac{1}{2}at^2$$

$$v = v_0 + at \Leftrightarrow 0 = v_0 + at \Leftrightarrow t = -\frac{v_0}{a}$$

$$\text{(a):} \qquad t = -\frac{v_0}{a} = -\frac{120\,\mathrm{km\,h^{-1}}}{-3\,\mathrm{m\,s^{-2}}} = 11.111\,\mathrm{s} \approx 11.1\,\mathrm{s}.$$

$$\text{(b):} \qquad \Delta x = v_0 t + \frac{1}{2}at^2 = 120\,\mathrm{km\,h^{-1}} \times 11.1\,\mathrm{s} + \frac{1}{2}\left(-3\,\mathrm{m\,s^{-2}}\right) \times (11.1\,\mathrm{s})^2$$

$$= 185.185\,\mathrm{m} \approx 185.2\,\mathrm{m}.$$

14. [1, 5] A body acquires an acceleration of magnitude $3.75\,\mathrm{m\,s^{-2}}$ when subjected to forces $\vec{F}_1 = (-2.00\vec{e}_x + 2.00\vec{e}_y)\,\mathrm{N}$ and $\vec{F}_2 = (5.00\vec{e}_x - 3.00\vec{e}_y)\,\mathrm{N}$.

   (a) What is the direction of the acceleration?

   (b) What is the mass of the body?

Solution:

$\vec{F}_R = \vec{F}_1 + \vec{F}_2 = [(-2.00 + 5.00)\vec{e}_x + (2.00 - 3.00)\vec{e}_y]\ \mathrm{N} = (3\vec{e}_x - \vec{e}_y)\,\mathrm{N}$

$\vec{F}_R = m\vec{a} : \tan\theta = \frac{F_{Ry}}{F_{Rx}} = \frac{1}{3} \Rightarrow \theta = \arctan\left(\frac{1}{3}\right) = 0.321751\ \mathrm{rad} = 18.4350\,^\circ \approx 18.4\,^\circ$

As this angle is in the fourth quadrant, the angle in relation to the positive semi-axis of the x's is: $-18.4\,^\circ$

That is, the net force and hence the acceleration form an angle of $-18.4\,^\circ$ with the positive semi-axis of x's.

$m = \frac{F_R}{a} = \frac{\sqrt{F_{Rx}^2 + F_{Ry}^2}}{a} = \frac{\sqrt{(3\,\mathrm{N})^2 + (-1.0\,\mathrm{N})^2}}{3.75\,\mathrm{m\,s^{-2}}} = 0.843274\,\mathrm{kg} \approx 0.84\,\mathrm{kg}.$

15. [0, 5] Calculate the gravitational force between two bodies of masses $m_1 = 2\,\mathrm{kg}$ and $m_2 = 600\,\mathrm{g}$ that lie at a distance of $d = 50\,\mathrm{cm}$.

Solution:

$$F = G\frac{m_1 m_2}{d^2} = \frac{\left(6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}\right)(2\,\mathrm{kg})(0.600\,\mathrm{kg})}{(0.50\,\mathrm{m})^2} = 3.2016 \times 10^{-10}\,\mathrm{N}.$$

16. [1, 5] A body of mass $5\,\mathrm{kg}$ is resting on a horizontal surface. The coefficient of static friction between the body and the surface is 0.40 and the coefficient of kinetic friction 0.30.

   (a) Determine the magnitude of the minimum force that causes the body starting to move?

   (b) Determine the magnitude of the minimum force that keeps the body in motion, once the body has started to move?

   (c) Determine the magnitude of the frictional force if we apply a horizontal force of $12\,\mathrm{N}$ on the body.

   (d) If the horizontal force is $50\,\mathrm{N}$, what is the magnitude of the frictional force?

Solution:

$F_{ae\,\mathrm{max}} = \mu_e mg = (0.40)(5\,\mathrm{kg})(9.8\,\mathrm{m\,s^{-2}}) = 19.6\,\mathrm{N}$

$F_{ac} = \mu_c mg = (0.30)(5\,\mathrm{kg})(9.8\,\mathrm{m\,s^{-2}}) = 14.7\,\mathrm{N}$

(a) F_{min} (that causes the body starting to move) $= F_{ae\,\mathrm{max}} = 19.6\,\mathrm{N}$

(b) F_{min} (that keeps the body in motion) $= F_{ac} = 14.7\,\mathrm{N}$

(c) $F = 12\,\text{N} \Rightarrow F_a = F = 12\,\text{N}$

(d) $F = 50\,\text{N} \Rightarrow F_a = F_{ac} = 14.7\,\text{N}$

17. [1, 5] Two small spheres which are considered material points, with masses of 1 kg and 3 kg are connected to one another by a rod with 1 m in length. Determine the position of the center of mass of the system, if the rod has:

(a) negligible mass;

(b) 0.4 kg, is homogeneous and has constant section.

Solution: *Let us assume that the rod is aligned with the x axis and that the sphere of mass* 1 kg is *at the origin of this axis.*

(a) $\vec{r}_{\text{CM}} = \frac{m_1\vec{r}_1+m_2\vec{r}_2}{m_1+m_2} = \frac{(1\,\text{kg})(0)+(3\,\text{kg})(\vec{e}_x)\,\text{m}}{1\,\text{kg}+3\,\text{kg}} = (0.75\vec{e}_x)\ \text{m}$
Alternative solution (using a lighter notation): $x_{CM} = \frac{m_1x_1+m_2x_2}{m_1+m_2} = \frac{(1\,\text{kg})(0)+(3\,\text{kg})(1\,\text{m})}{(1+3)\,\text{kg}} =$ 0.75 m.

(b) $\vec{r}_{\text{CM}} = \frac{m_1\vec{r}_1+m_2\vec{r}_2+m_3\vec{r}_3}{m_1+m_2+m_3} = \frac{(1\,\text{kg})(0)+(3\,\text{kg})(\vec{e}_x)\,\text{m}+(0.4\,\text{kg})(0.5\vec{e}_x)\,\text{m}}{1\,\text{kg}+3\,\text{kg}+0.4\,\text{kg}} = (0.73\vec{e}_x)\ \text{m}$
Alternative solution (using a lighter notation):
$x_{CM} = \frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3} = \frac{(1\,\text{kg})(0)+(3\,\text{kg})(1\,\text{m})+(0.4\,\text{kg})(0.5\vec{e}_x)\,\text{m}}{(1+3+0.4)\,\text{kg}} = 0.73\,\text{m}.$

18. [0, 5] Find the pressure increase in the fluid in a syringe when a nurse applies a force of 42 N to the syringe's circular piston, which has a redius of 1, 1 cm.

Solution:
$p = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{42\,\text{N}}{\pi(0.011\,\text{m})^2} = 110488\,\text{Pa} \approx 1.1 \times 10^5\,\text{Pa}.$

19. [1] Find the pressure at a depth of 15 m in seawater, assume that the density of seawater is about 1.25 kg/ l.

Solution: $p = p_0 + \rho g h = 1.013\times10^5\,\text{Pa}+1.25\times10^3\,\text{kg}\,\text{m}^{-3}\times9.8\,\text{m}\,\text{s}^{-2}\times15\,\text{m} = 2.85050\times10^5\,\text{Pa}.$

20. [1, 5] Water is moving with a speed of $5.0\,\text{m}\,\text{s}^{-1}$ through a pipe with a cross-sectional area of $4.0\,\text{cm}^2$. The water graduall descends 7 m as the pipe cross-sectional area increases to $8.0\,\text{cm}^2$.

(a) What is the speed at the lower level?

(b) If the pressure at the upper level is 2×10^5 Pa, what is the pressure at the lower level?

Solution:

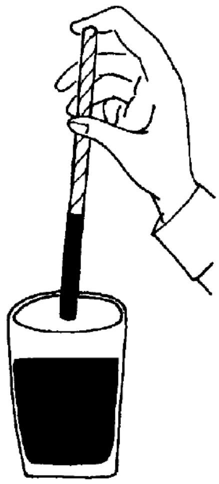
(a) Continuity equation: $A_1v_1 = A_2v_2 \longrightarrow A_1v_1 = 2A_1v_2 \longrightarrow v_2 = \frac{v_1}{2} = \frac{5.0}{2} = 2.5\,\text{m/s}$

(b) Bernoulli equation: $p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$
$p_2 = p_1 + \rho\frac{1}{2}\left(v_1^2 - v_2^2\right) + \rho g\left(h_1 - h_2\right)$
$p_2 = p_1 + \rho\frac{1}{2}\left(v_1^2 - v_2^2\right) + \rho g\left(h_1 - h_2\right)$
$p_2 = 2\times10^5\,\text{Pa}+(1000\,\text{kg}\,\text{m}^{-3})\,\frac{1}{2}\left(\left(5.0\,\text{m}\,\text{s}^{-1}\right)^2 - \left(2.5\,\text{m}\,\text{s}^{-1}\right)^2\right)+(1000\,\text{kg}\,\text{m}^{-3})\,(9.8\,\text{m/s}^2)\,(7\,\text{m})$
$= 2 \times 10^5\,\text{Pa} + 9375.0\,\text{Pa} + 68600.0\,\text{Pa} = 277975\,\text{Pa} \approx 2.8 \times 10^5\,\text{Pa}.$

21. [1, 5] In the laboratory classes of this discipline we have made an improvised dropper. Briefly explain what was done and explain also using a physical explanation why it works.

Solution:

Improvised dropper



(a) Color a bit of water with vegetable dye.

(b) Place a paper or glass straw in a glass with the colored water.

(c) Suck up a little of the water into the straw.

(d) Then hold your finger across the top of the straw and pull the straw out of the liquid. What happens?

(e) Then remove your finger from the top of the straw for a brief period of time.

You will see that: While your finger covers the top of the straw, the liquid remains in the straw. When you remove the finger, the water flows out.

Expanation: Rising up the straw with the finger covering the top of the straw, a small amount of liquid leaves the straw, which makes the air pressure inside the straw to decrease (because the volume available to the air has increased). The air pressure at the bottom of the straw is greater and prevents the fluid from flowing from the interior of the straw.